Attempts to understand variational phase-field fracture for nearly, incompressible solids with experimental data

Thomas Wick

Leibniz University Hannover (LUH), Germany
Institute for Applied Mathematics (IfAM)
AG Wissenschaftliches Rechnen (GWR)

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Collaborators in this work

- Katrin Mang (Leibniz University Hannover, Germany) (**Here!**)
- Nils Hendrig Kröger (DIK - German Kautschuk Institute, Germany)
- Winnifried Wollner (Univ. Darmstadt, Germany)
- Miriam Walloth (Univ. Darmstadt, Germany)
- Timo Heister (Clemson Univ., USA)
- Dominik Wick (EJOT, middle-sized company in fastening technologies, Germany)
- Ralph Jörg Hellmig (EJOT, Germany)
- Hans-Jürgen Christ (Univ. Siegen, Germany)
- Meng Fan (Leibniz University Hannover/China University of Petroleum, Beijing)
- Jin Yan (China University of Petroleum, China)

Overview

Motivation (I)

Baseline:

- Numerous studies in calculus of variations and engineering on variational approaches (phase-field) for fracture (Francfort/Marigo, 1998; Bourdin/Francfort/Marigo, 2000/2008; Bourdin 2007; Chambolle at al. 2008; Amor et al. 2009; Miehe/Hofreither/Welschinger, 2010/2010; Burke/Ortner/Süli, 2010, ...)
- Some are qualitatively useful; but quantitative studies are rarely about to find ... in particular in few of nearly-incompressible solids (see also talks from yesterday L. Anand and O. Lopez-Pamies)
- Classical numerical tasks such as $h \to 0$ often yield non-satisfactory findings. Why?

$\rightarrow$ interaction with several regularization parameters (phase-field, crack irreversiblity, ...)

$\rightarrow$ various solid mechanics models for energy (stress) splittings (Amor et al. 2009; Miehe et al. 2010; Zhang et al. 2017; Strobl/Seelig, 2015; Steinke/Kaliske, 2019; Bryant/Sun, 2018; Freddi/Royer-Carfagni, 2011, ...)

Motivation (II)

Before we address nearly-incompressible materials,
we briefly review some of our results for compressible solids.

A positive example: Sneddon's 2D/3D stationary pressurized fracture

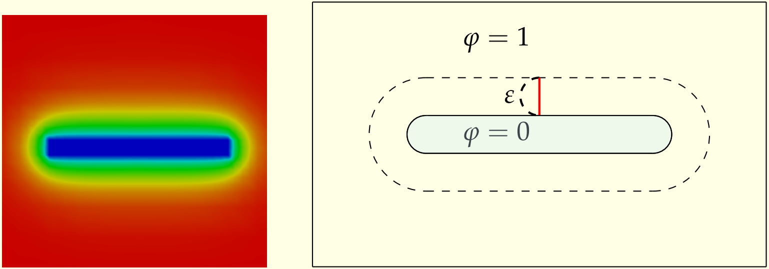

Figure: Fracture representation using the phase field φ. The inner blue region indicates the crack with $\varphi = 0$ and the red region indicates the unbroken zone where $\varphi = 1$. We have a diffusive zone in between (green region).

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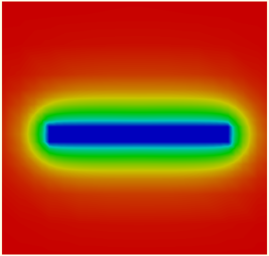

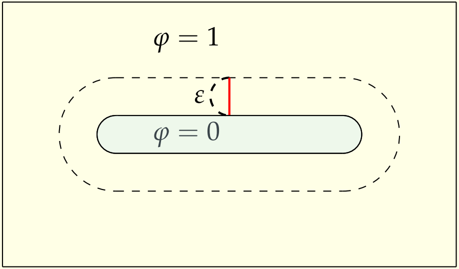

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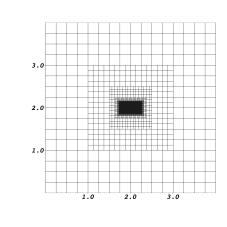
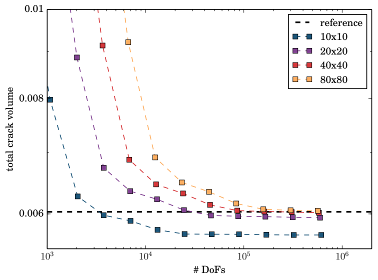
- Analytical derivations in Sneddon/Lowengrub 1969, Section 2.4 (2D), Section 3.3 (3D); fracture does not propagate but only varies in its width;
- Goal: Compute a **goal functional value**. Let's say the total volume of the fracture. Is it possible to obtain numerically computed values that coincide for $h \to 0$ with manufactured solutions? (Standard task in numerics!)

Sneddon's 2D/3D pressurized fracture: challenges

- No. 1: Phase-field smears out the fracture;
- No. 2: It is crucial how φ is initialized;
- No. 3: It is crucial how $\varepsilon - h$ is chosen;
- No. 4: Moreover, very, very fine meshes around the crack path are necessary! (Adaptive mesh refinement).



2D results for TCV (total crack volume): error w.r.t. manufactured solution [1]



- Total crack volumes:

$$TCV_{h,\epsilon} = \int_\Omega u \cdot \nabla \varphi dx,$$

$$TCV_{2d} = \int_x 2u_y(x)dx = \frac{2\pi p l_0^2(1-\nu^2)}{E},$$

$$TCV_{3d} = \int_x \int_y 2u_z(x,y)dxdy = \frac{16 p l_0^3(1-\nu^2)}{3E}$$

where $l_0 = 1.0$, $E = 1.0$, $p = 1e-3$, $\nu = 0.2$.

Domain	TCV	Rel. err.
10x10	5.6942E-03	5.60%
20x20	5.9393E-03	1.53%
40x40	6.0014E-03	0.51%
80x80	6.0383E-03	0.11%
Ref.	6.0319E-03	

[1] Heister/Wick; 2018, PAMM; see also Bourdin et al. 2012, SPE

Length-scale refinement

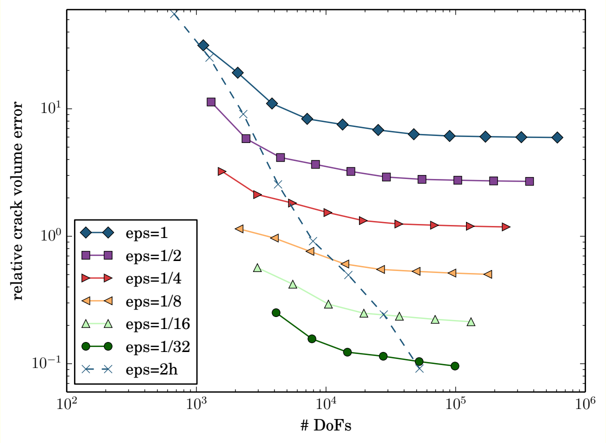
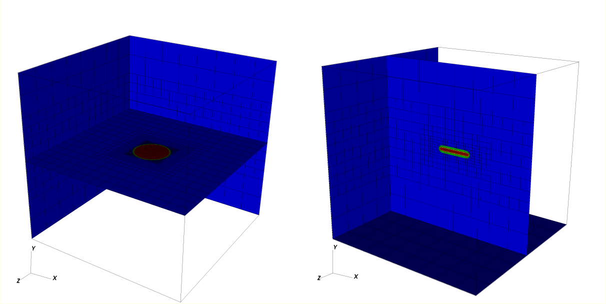


Figure: Linear convergence in the length-scale ε.

3D results for TCV

ref	$10l_0/h$	# DoFs	eps	TCV	Error
1	2	500	1.7321E+01	7.4519E-02	1355.44%
⋮					
8	256	946,852	1.3532E-01	5.8082E-03	13.44%
9	512	2,938,012	6.7658E-02	5.3764E-03	5.01%
10	1024	9,318,916	3.3829E-02	5.2131E-03	1.82%
11	2048	30,330,756	1.6915E-02	5.1567E-03	0.72%
12	4096	100,459,828	8.4573E-03	5.1352E-03	0.30%
Sneddon (exact)				5.1200E-03	



Parallel scalability for Sneddon's 2D test [2]

- Single fracture
- Pressure force acts on fracture boundaries
- Local mesh refinement
- Parallel computing

		NP							
ref	Dofs	16	32	64	128	256	512	1024	2048
5	198'147	19	19	19	18	18	18	17	19
6	789'507	24	23	25	24	25	23	24	23
7	3'151'875	33	27	29	28	27	27	25	37
8	12'595'203	-	-	31	31	33	32	30	32
9	50'356'227	-	-	43	43	44	48	40	52

Table: Number of GMRES iterations of a single Newton step for the Sneddon 2d test with global refinement. Iterations are nearly independent of problem size (h) and number of processors NP. The relative residual is 1e-8.

[2] Heister/Wick; PAMM, 2018

Second example: damage and fatigue of screws (I)

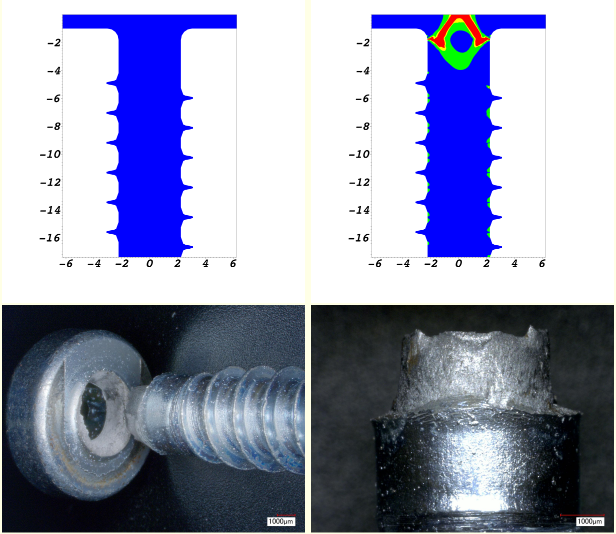
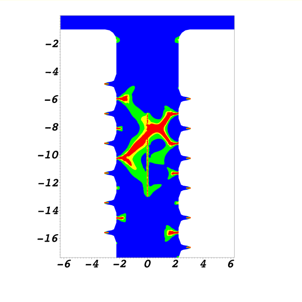


Figure: Industrial collaboration: Uniaxial tension and crack nucleation at points with highest stresses. Experimental data from D. Wick (EJOT, Germany).

Damage and fatigue of screws (II) [3]



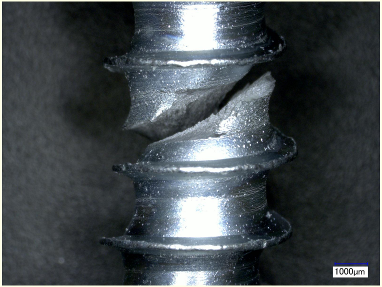


Figure: Uniaxial tension and crack nucleation at points with highest stresses. Experimental data from D. Wick (EJOT, Germany).

- Qualitatively the EJOT company was happy about the crack paths! (What they considered as their quantity of interest (QoI)!)

[3] D. Wick, T. Wick, R.J. Helmig, H-J. Christ; Comput. Mater. Sci July 2015