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Bond-based heterogeneous material model

- Observe that in equilibrium with $b \equiv 0$ and fixed stress σ_0,

$$u_0(x) = \int_0^x \frac{\sigma_0}{E(z)}\, dz.$$

- From this compute the smoothed displacements:

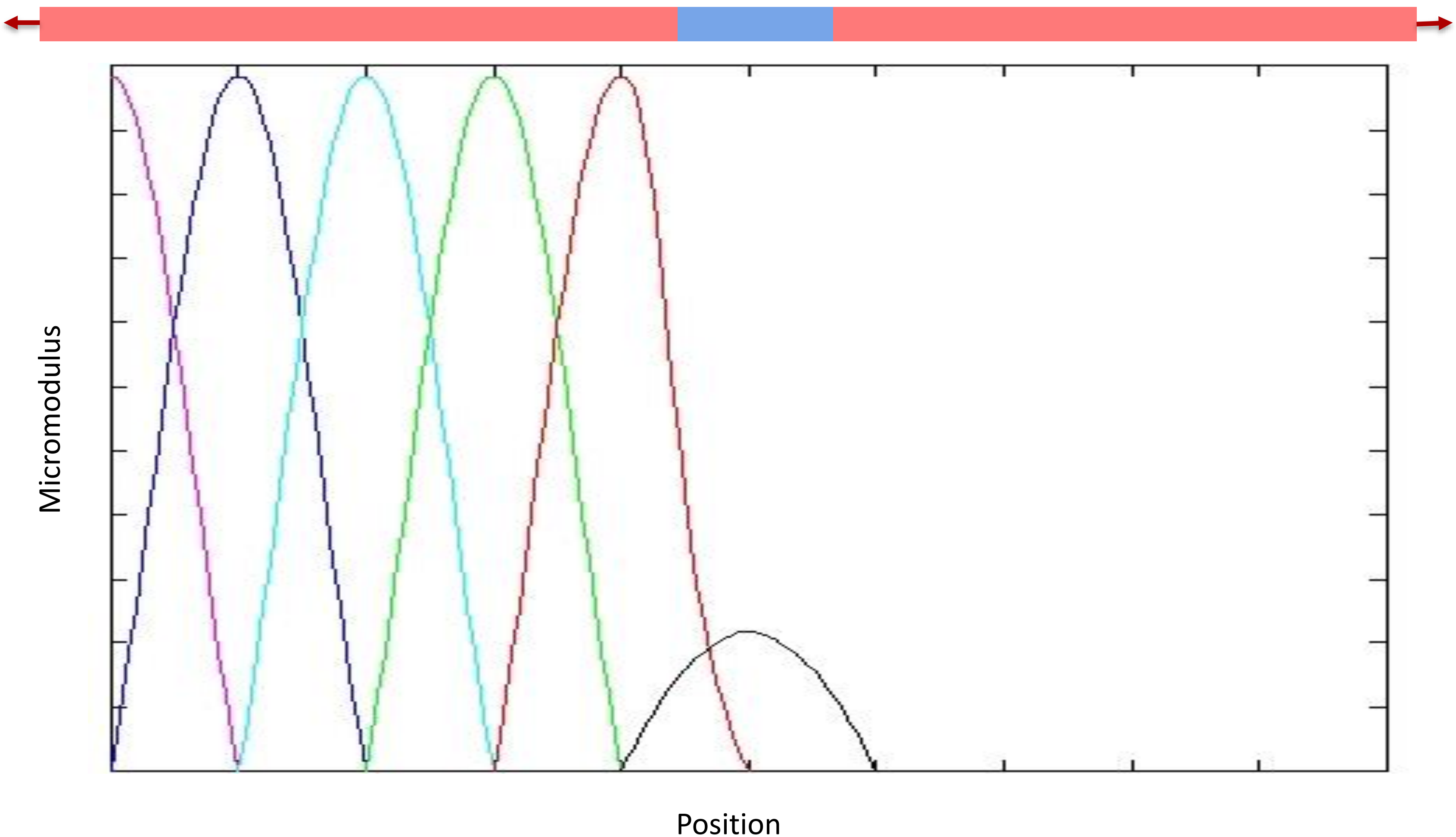
$$\bar{u}(x) = \int_{-\epsilon}^{\epsilon} w(\zeta) u_0(x+\zeta)\, d\zeta.$$

- Define a nonlocal material model by (omit t):

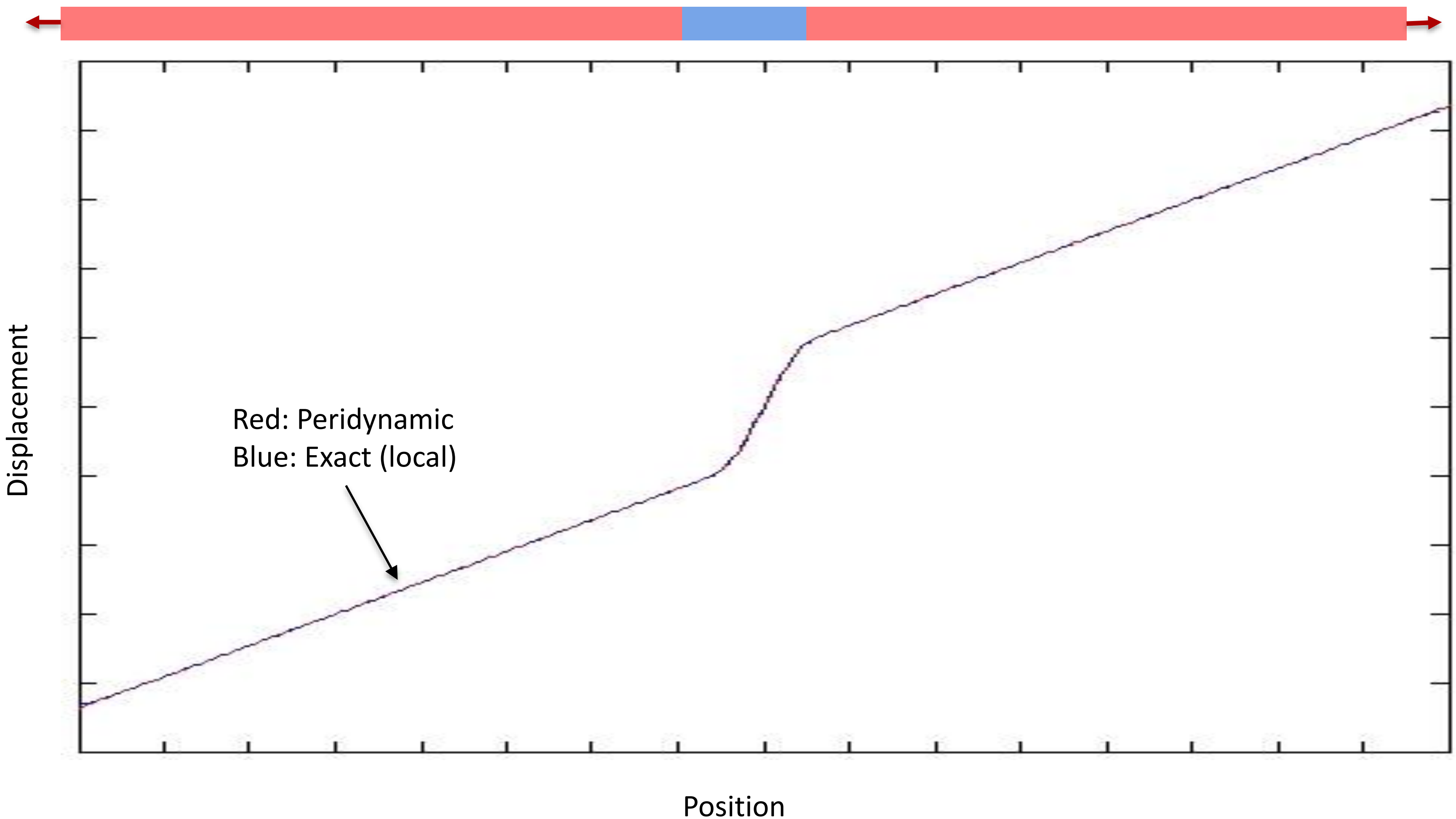
$$f(q,x) = C(q,x)(\bar{u}(q) - \bar{u}(x)), \qquad C(q,x) := \frac{\sigma_0 w'((q-x)/2)}{\bar{u}_0(q) - \bar{u}_0(x)}.$$

- This exactly reproduces the local result for equilibrium with $b \equiv 0$.
- (But not in general.)

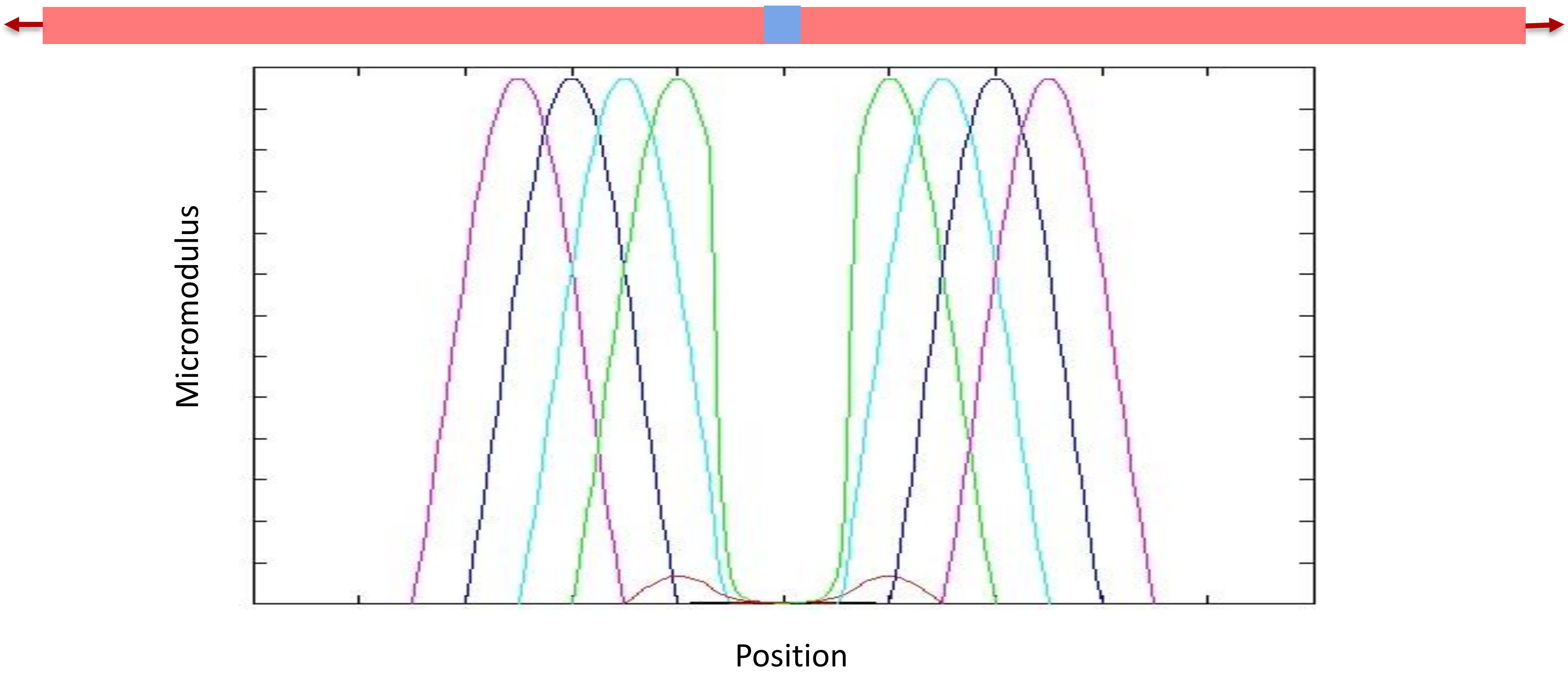
Bar with a soft spot: Micromodulus



Bar with a weak spot: Displacement



Bar with a very weak spot: Micromodulus shows broken bonds



- The heterogeneous peridynamic material model zeroes out the micromodulus for bonds crossing the crack.
- Bond breakage!

What the preceding analysis shows

- Using smoothed displacements results in a nonlocal evolution law.
- This evolution law is peridynamics provided a material model in terms of $\bar{u}$ is defined.
- The micromodulus is determined by:
  - The small-scale (local) material model and heterogeneity.
  - The smoothing function w.
- A nonlocal concept of damage (bond breakage) emerges naturally when the original problem contains a crack.

A hint of unexpected behavior

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- Recall

$$\bar{u}(x) = \int w(x-p)u(p)\, dp.$$

- Fourier transform of any function v:

$$v^*(k) = \mathcal{F}\{v(x)\} = \int_{-\infty}^{\infty} e^{-ikx} v(x)\, dx.$$

- Convolution theorem

$$\bar{u}^* = w^* u^*$$

so that formally we can derive the small-scale displacements from any given $\bar{u}$:

$$u(x) = \mathcal{F}^{-1}\left\{\frac{\bar{u}^*}{w^*}\right\}.$$

A hint of unexpected behavior, ctd.

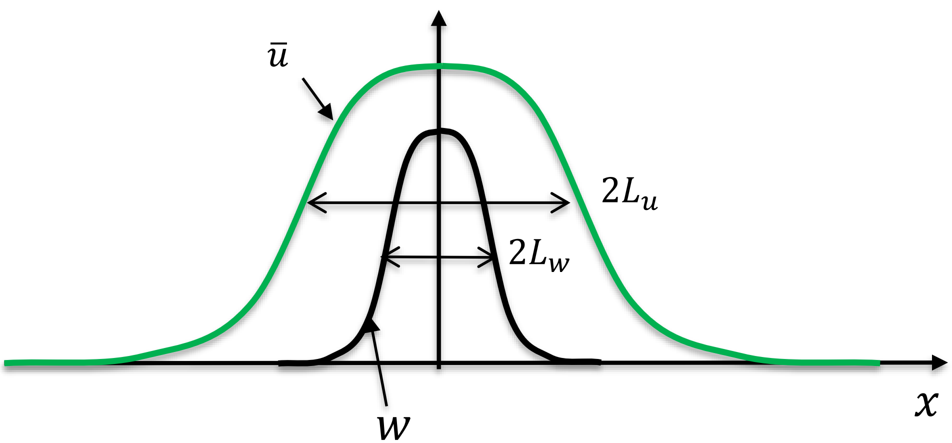
- Can we arbitrarily prescribe $\bar{u}$?
- Suppose w and $\bar{u}$ are both Gaussians:

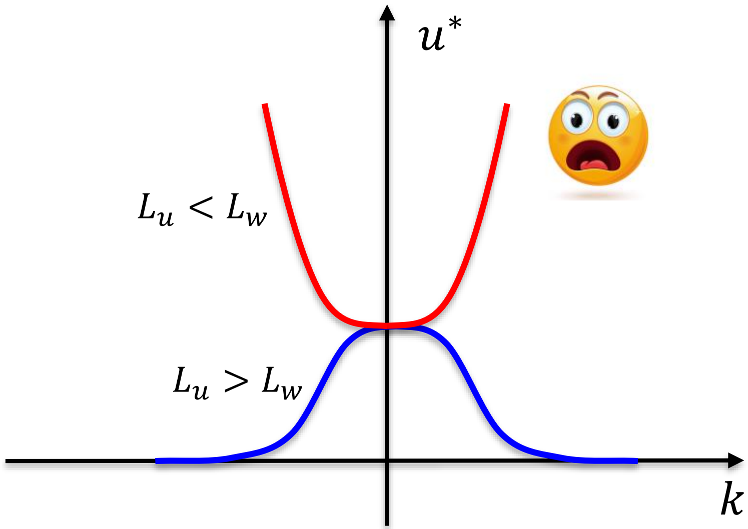
$$\bar{u}(x) = e^{-(x/L_u)^2}, \qquad w(x) = e^{-(x/L_w)^2}.$$

- Then

$$u^*(k) = \frac{\bar{u}^2(k)}{w^*(k)} = \sqrt{\frac{L_u}{L_w}}\, e^{\pi^2(L_w^2 - L_u^2)k^2}$$

- Bad news if $L_u < L_w$!



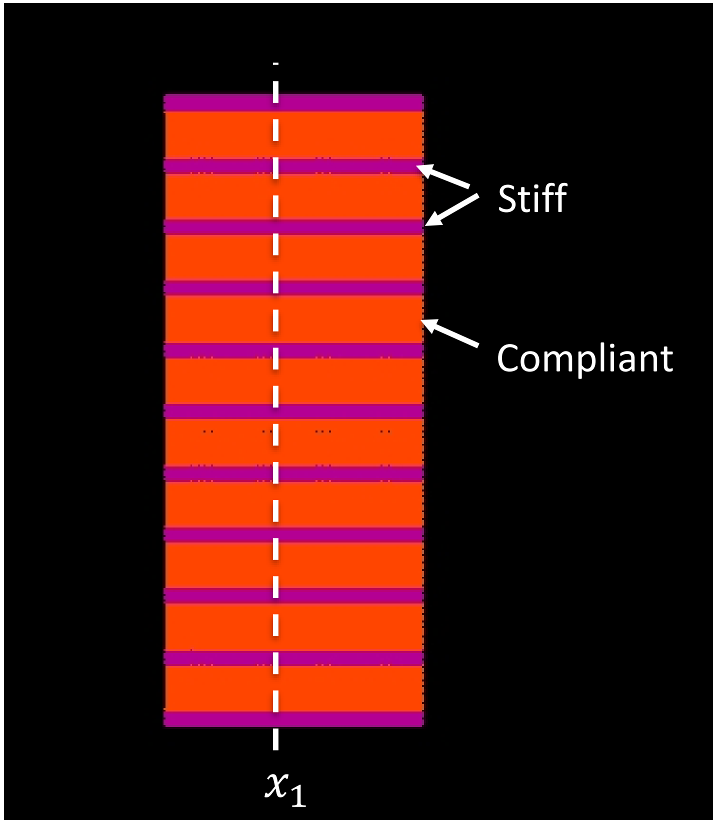


Can nonlocality be observed experimentally in elastostatics?

- Consider a 2D composite composed of alternating layers of stiff and compliant material.
- Smoothed DOF is the average x displacement along a vertical line.

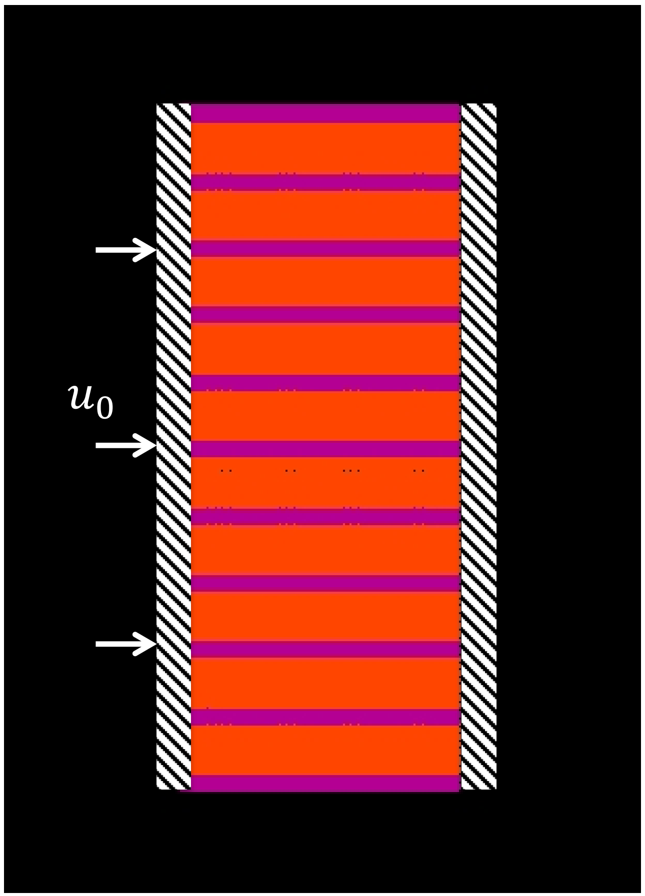
$$\bar{u} = \frac{1}{L}\int_0^L u_1 \, dx_2$$

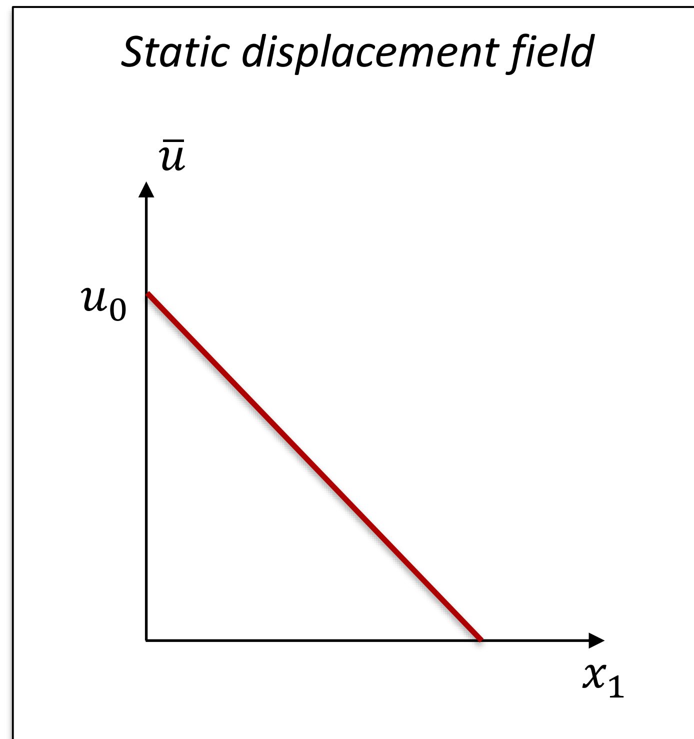
- We will examine "seemingly" 1D deformations.



Static Dirichlet problem for a composite

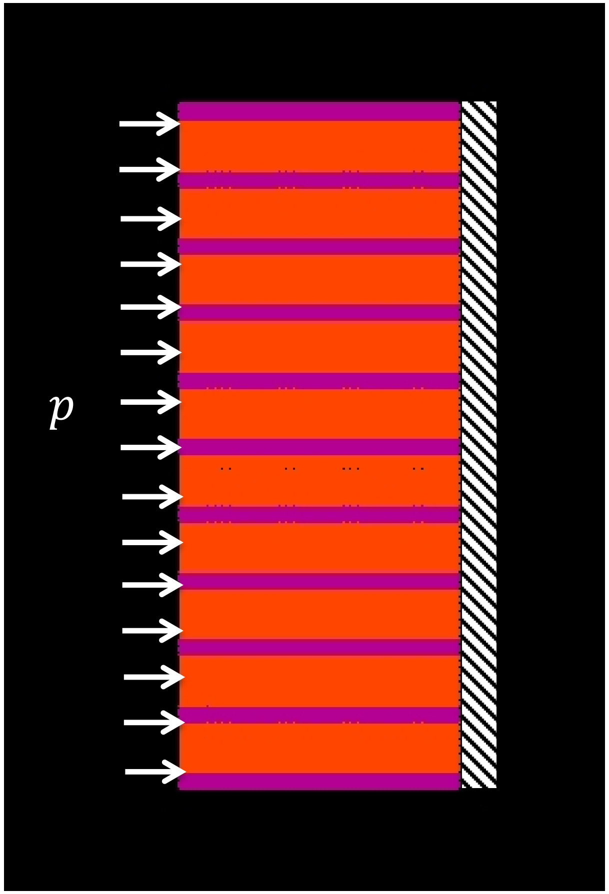
- Solve for the 2D displacements in the local theory.
- Both phases deform the same way.
- No surprises (yet).





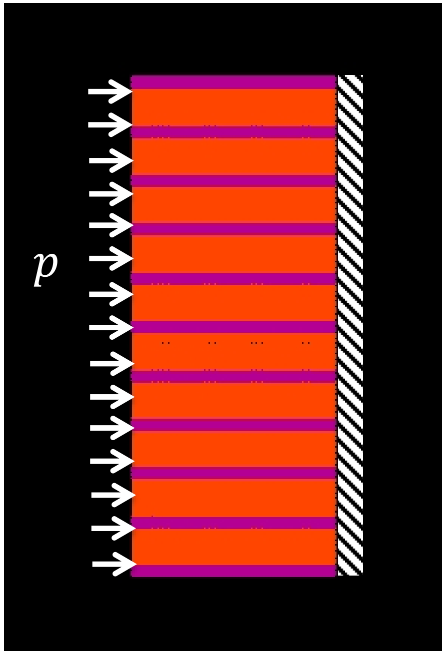
Now consider a mixed Dirichlet/Neumann static problem

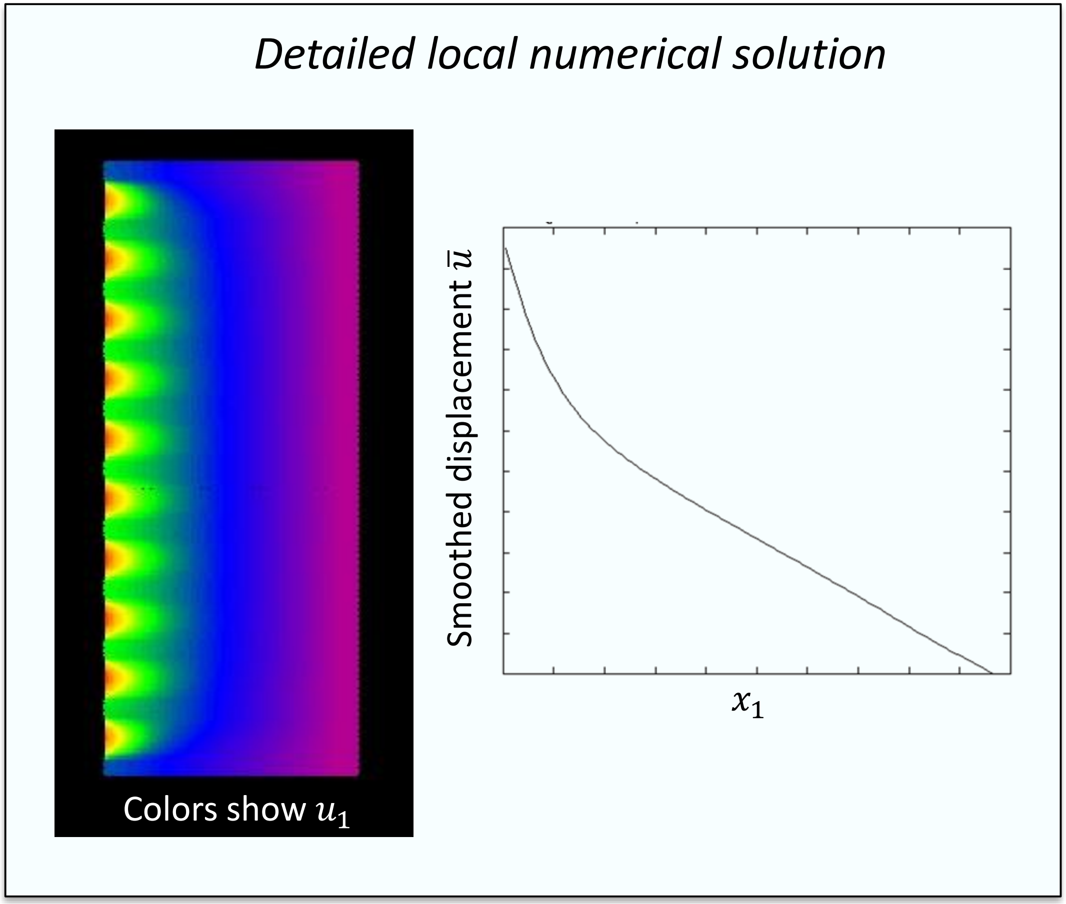
- Apply a constant traction p along the left surface.
- Still using 2D local theory.
- Should we still expect $\bar{u}$ to vary linearly with x_1?



Smoothed DOFs show interesting features

- A detail computational model shows complex behavior near the left edge.
- Smoothing this solution results in nonlinear $\bar{u}(x_1)$.

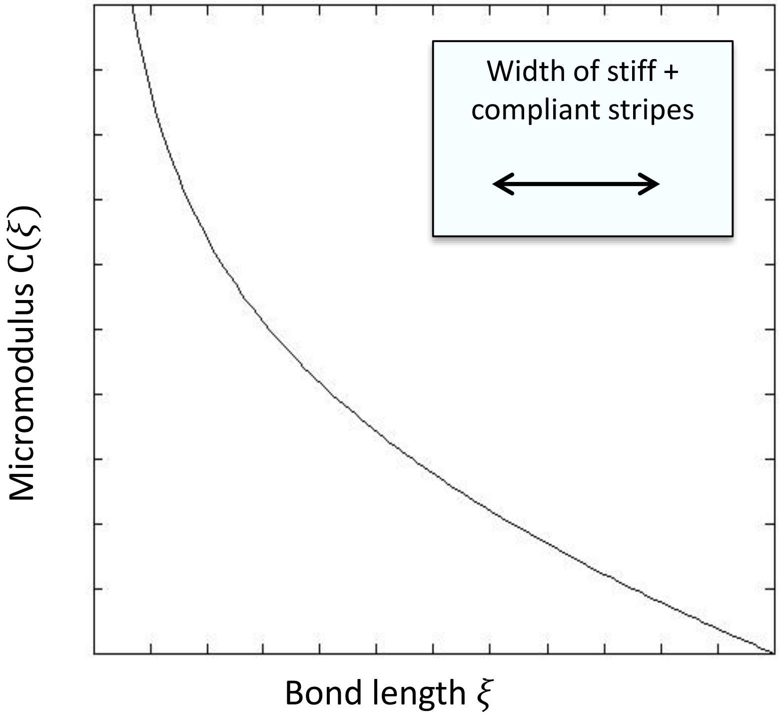




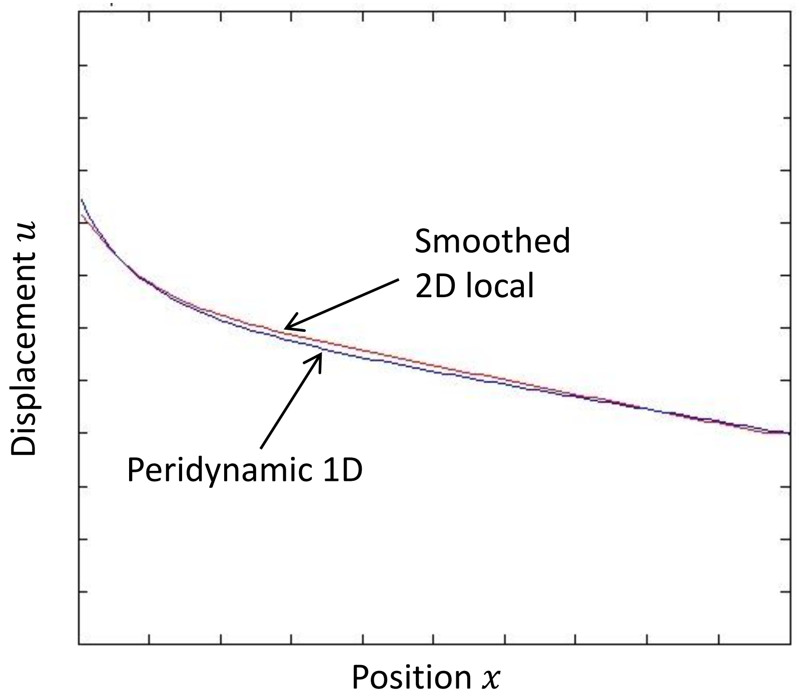
Nonlocality helps reproduce response near loaded boundary

- Tune a 1D peridynamic microelastic material model.
- Try to reproduce the behavior seen in the detailed 2D local solution..

Peridynamic material model

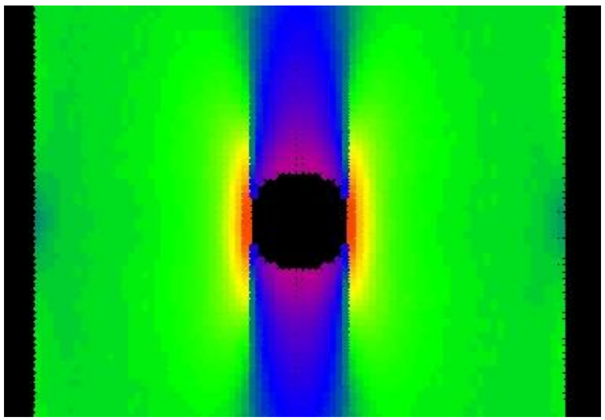


Predicted displacement

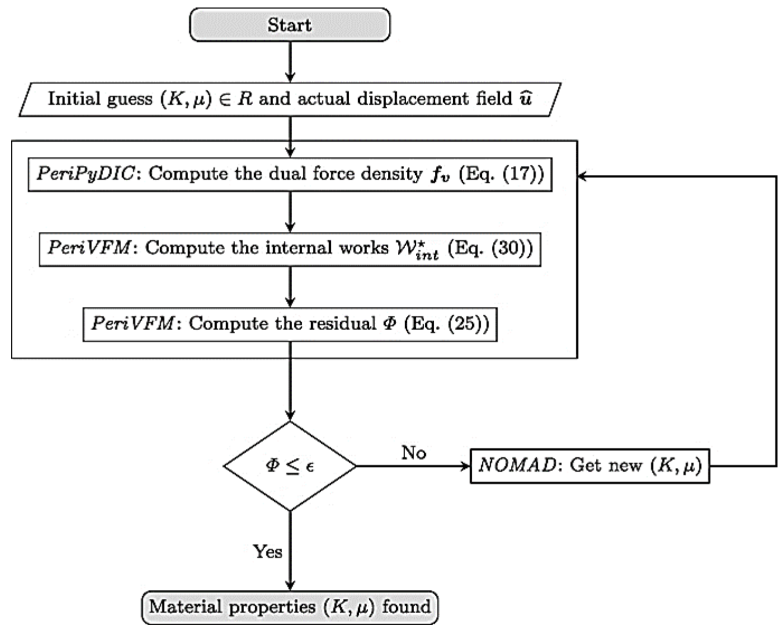


Nonlocal material parameters can be derived from static full-field data

- Digital image correlation (DIC).
- Virtual field method (VFM).
- Electronic speckle pattern interferometry (ESPI).



Strain concentration in a composite is influence by nonlocal length scales

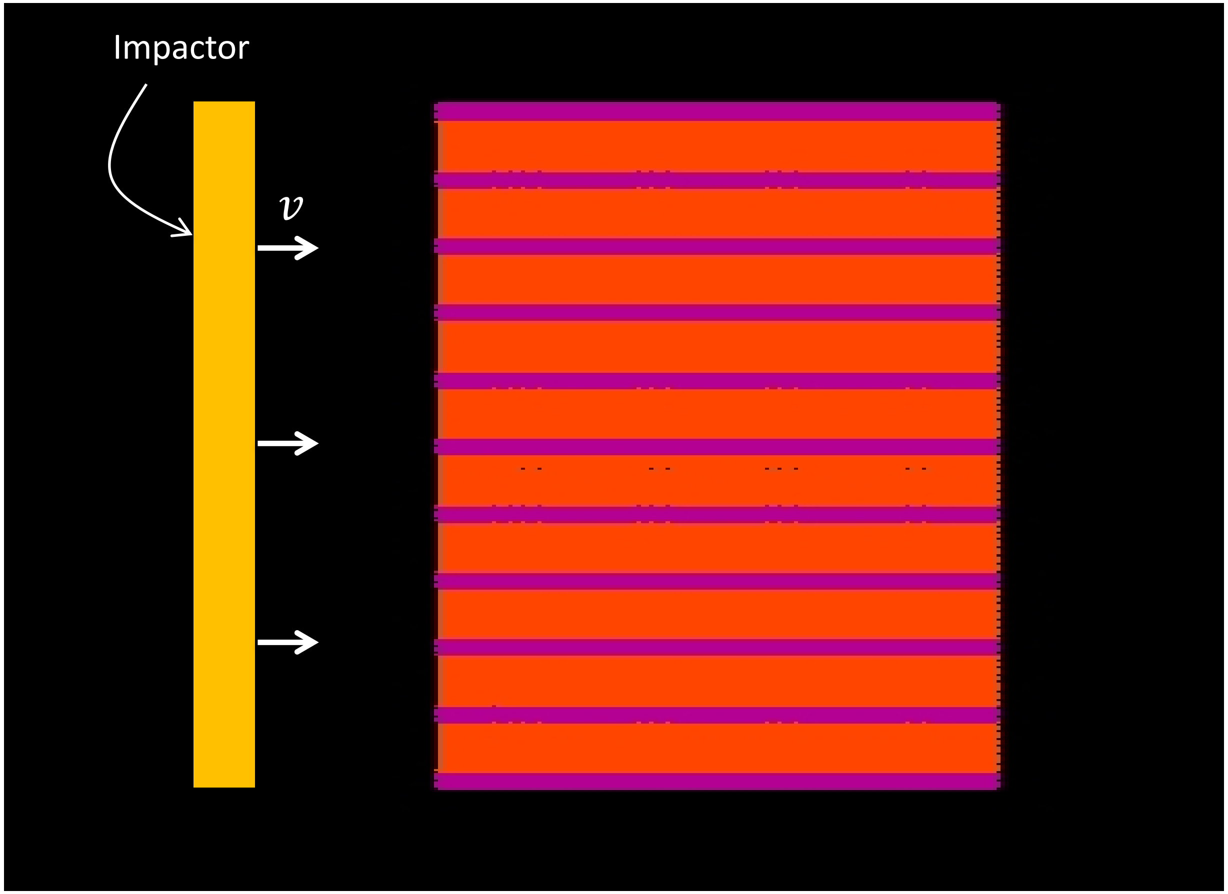


Flow chart for obtaining PD parameters by VFM*.

- L. Toubal, M. Karama, & B. Lorrain, *Composite structures*, (2005).
- D. Turner, B.Van Bloemen Waanders, & M. Parks *J. Mechanics of Materials and Structures* (2015).
- D. Turner, *J. Engineering Mechanics* (2015).
- *Delorme, R., Diehl, P., Tabiai, I. et al., *J Peridyn Nonlocal Model* (2020)

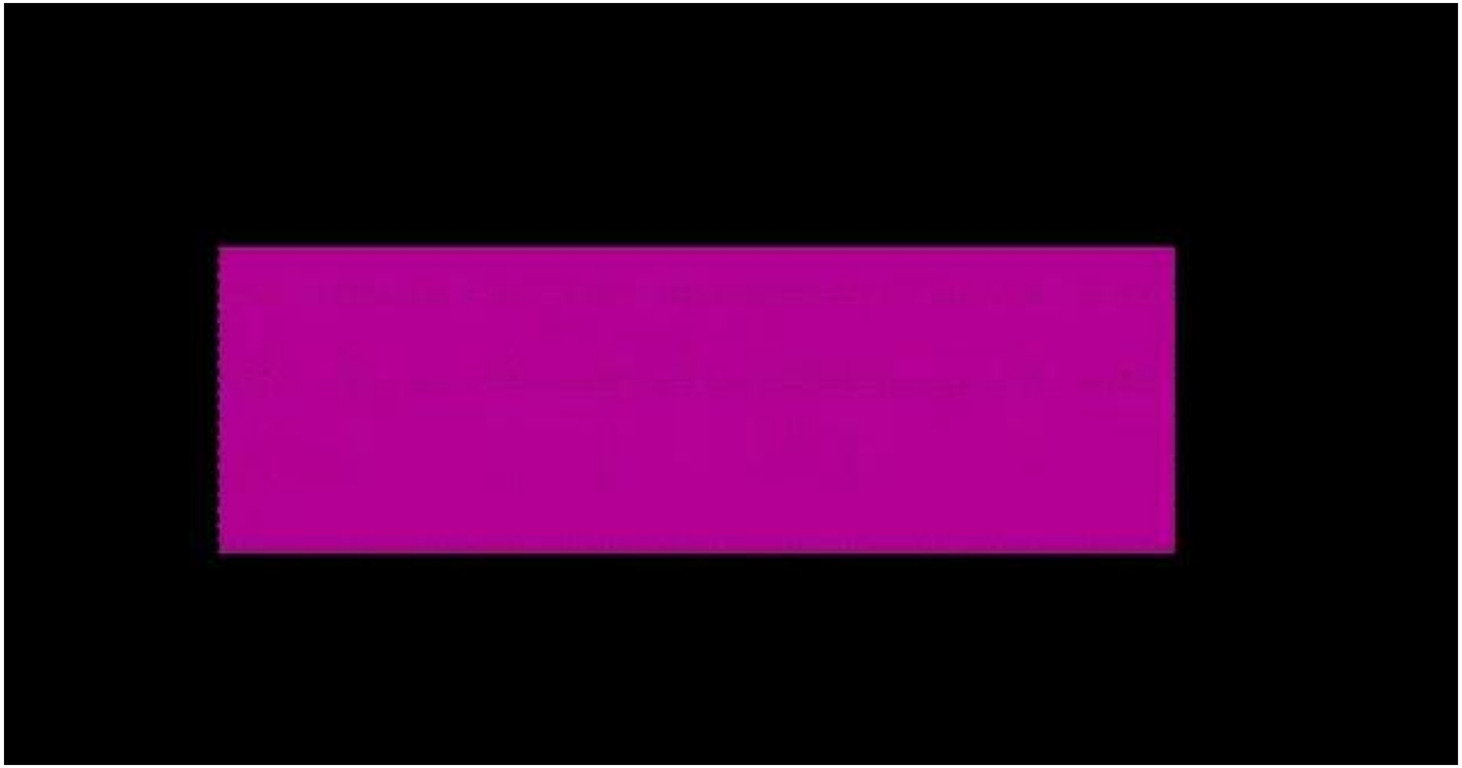
Dynamics: impact problem

- Impactor strikes the composite edge-on.



Dynamics: impact problem video

- Detailed 2D local simulation.
- Complex wave structure is created in the composite.



Colors show maximum principal strain

Dynamics: impact problem

- Detailed 2D local simulation.
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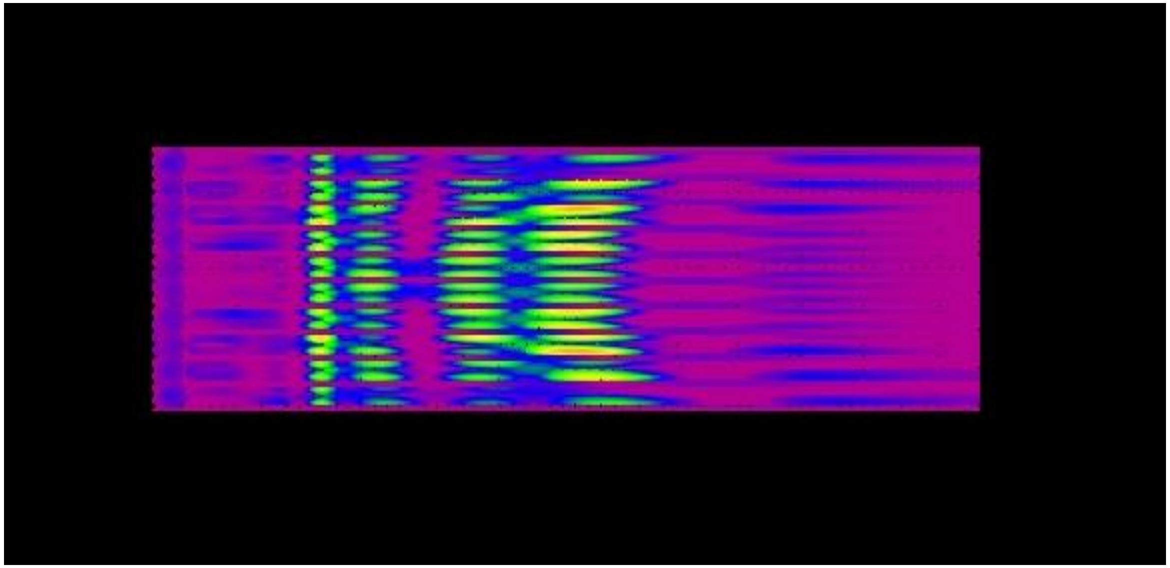
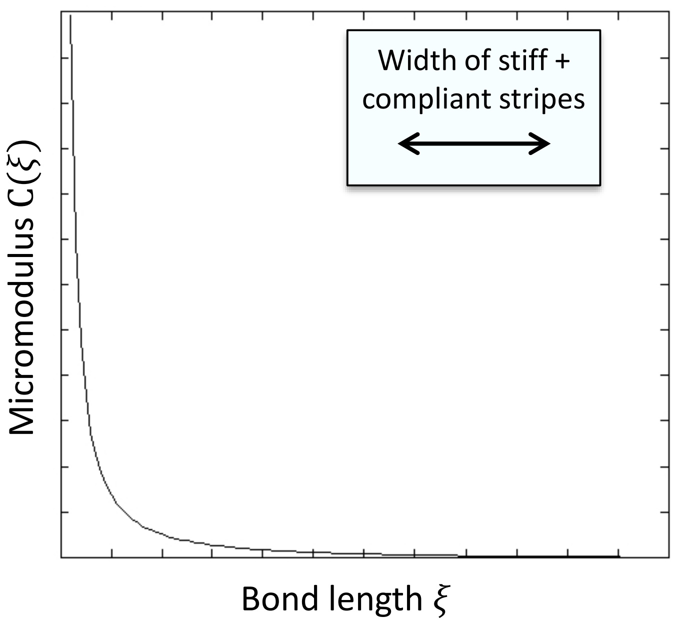


Colors show maximum principal strain

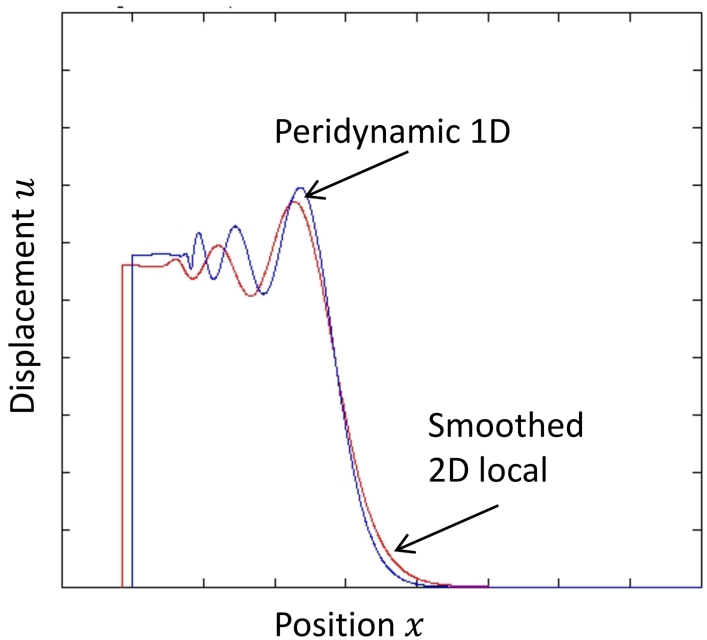
Nonlocality helps predict the dispersive nature of waves in the composite

- After smoothing the displacement along vertical lines, the complex wave structure is manifested as dispersion.
- A 1D peridynamic model (after tuning of the micromodulus) reproduces some of these features.

Peridynamic material model

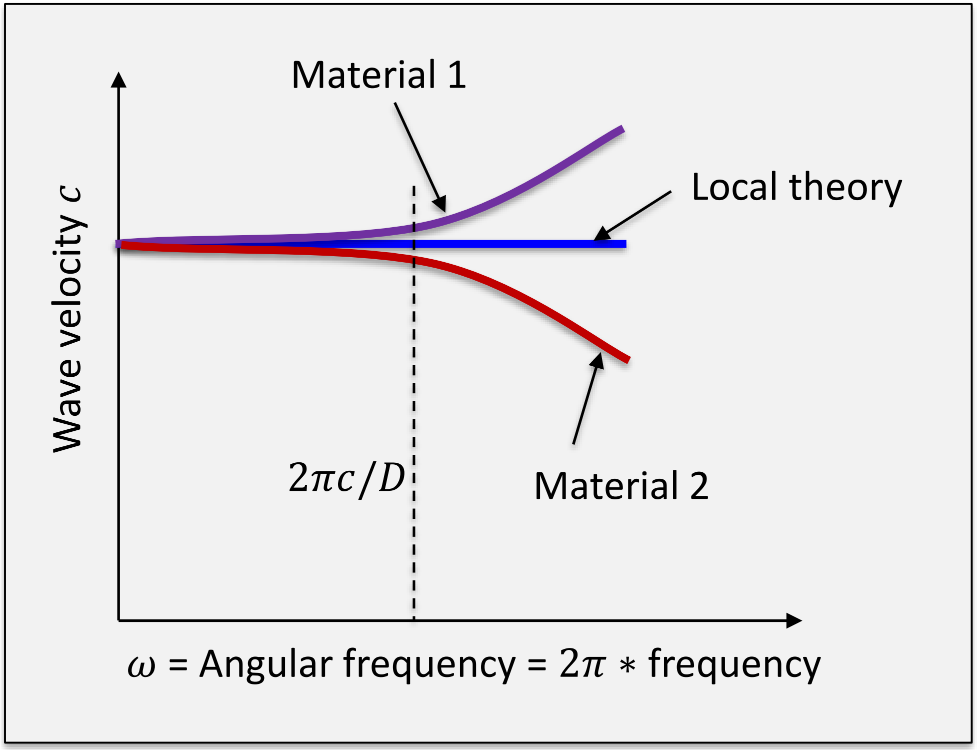


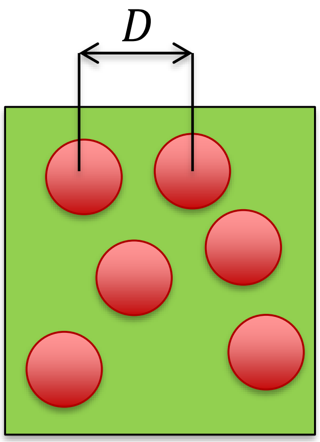
Wave structure



Wave dispersion

- All real solids exhibit dispersion for sufficiently short wavelengths.
- The wavelength depends on the microstructure and composition.
  - Dispersion starts to appear for **wavelengths < microstructure size**.
  - This implies that nonlocality is required to predict dispersion.





Microstructure

Wave dispersion in linear peridynamics

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- Equation of motion with $b \equiv 0$:

$$\rho \ddot{u}(x,t) = \int_{-\delta}^{\delta} C(\xi)(u(x+\xi,t) - u(x,t))\, d\xi$$

- Look for plane wave solutions of the form

$$u(x,t) = e^{i(kx-\omega t)}$$

where k=wavenumber and ω=angular frequency.

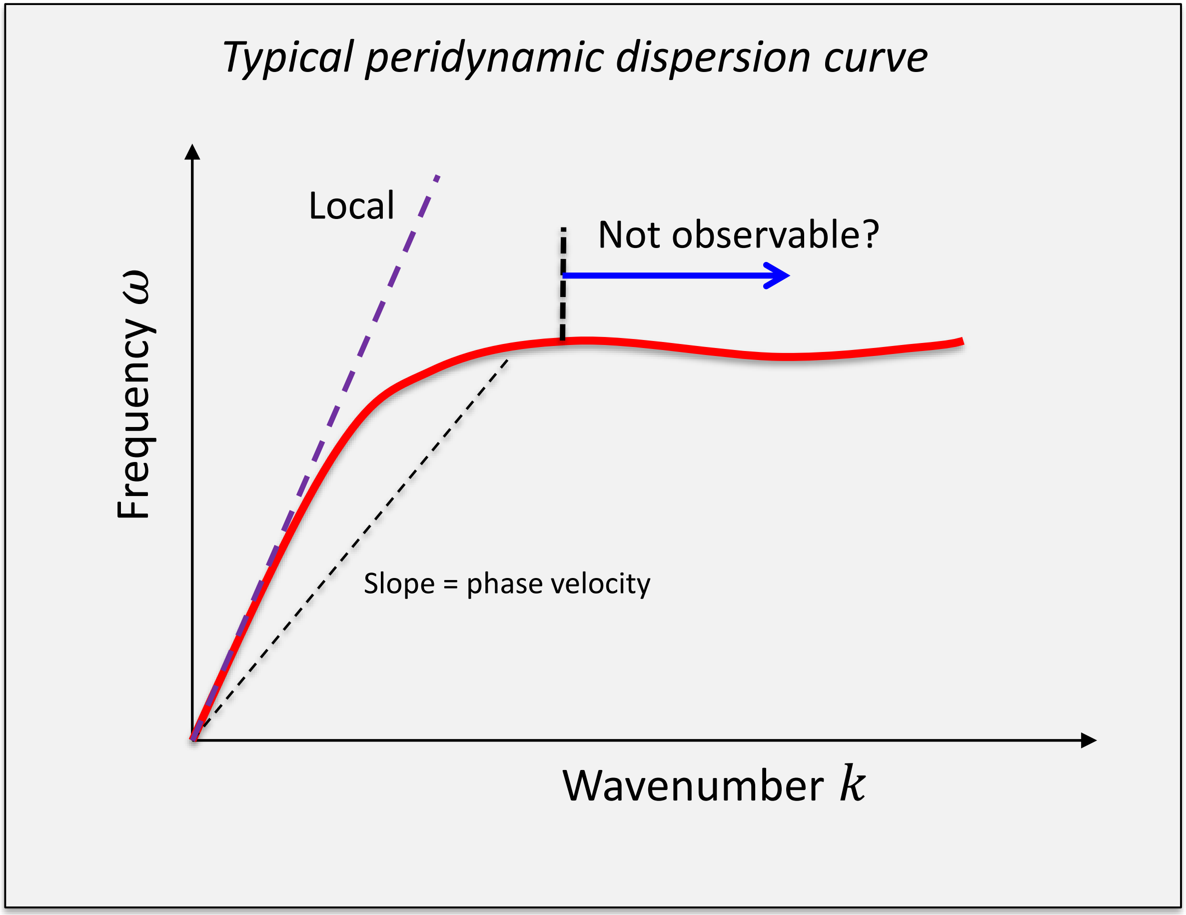
- Condition on ω and k:

$$-\rho\omega^2 = \int_{-\delta}^{\delta} C(\xi) e^{ik\xi}\, d\xi - P, \qquad P := \int_{-\delta}^{\delta} C(\xi)\, d\xi$$

- or in terms of the Fourier transform $C^* = \mathcal{F}\{C\}$,

$$\rho\omega^2(k) = P - C^*(k)$$

Wave dispersion in linear peridynamics



- S. N. Butt, J. J. Timothy, & G. Meschke, *Computational Mechanics* (2017).
- V. S. Mutnuri, USNCCM15 presentation (2019).

Finding peridynamic material properties from measured dispersion data

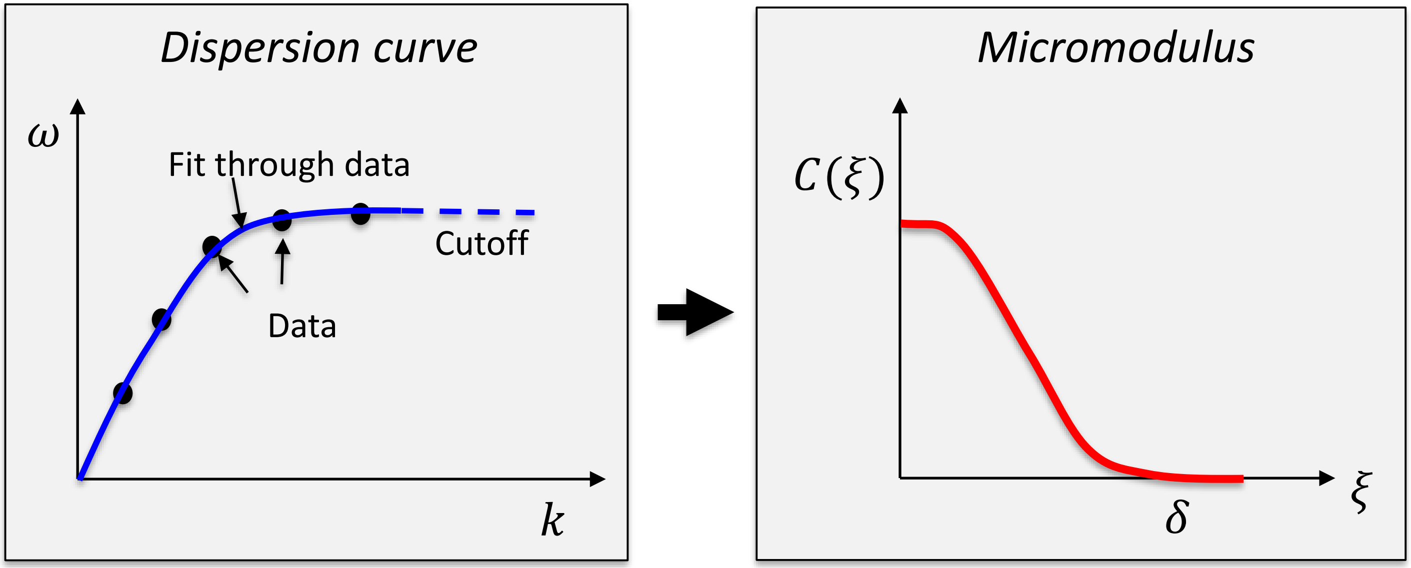
- We found

$$\rho\omega^2(k) = P - C^*(k).$$

- Given measured $\omega_{exper}(k)$, formally solve

$$C(\xi) = \mathcal{F}^{-1}\{P - \rho\omega^2_{exper}(k)\}$$

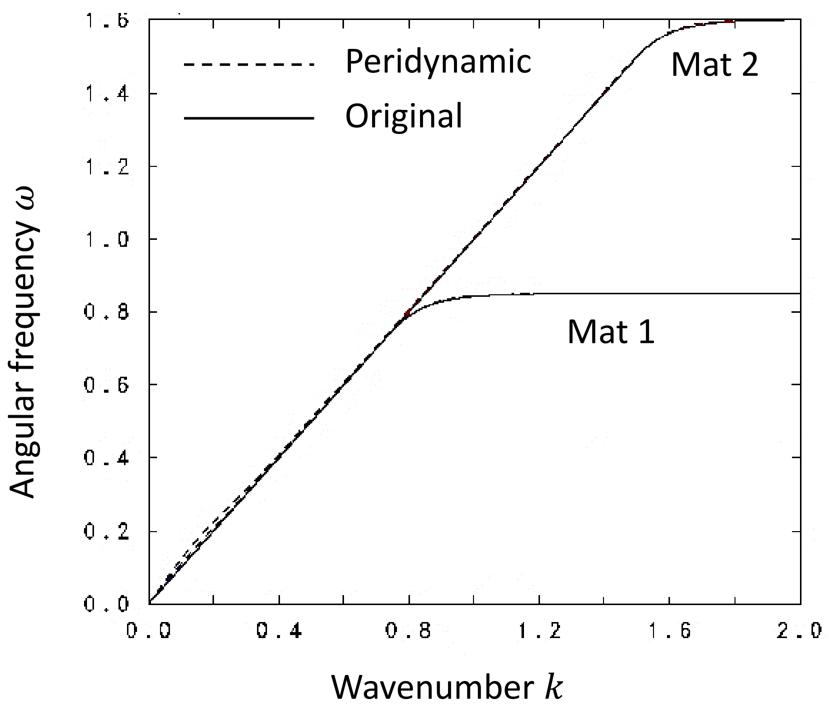
(requires data to be cut off for large k).

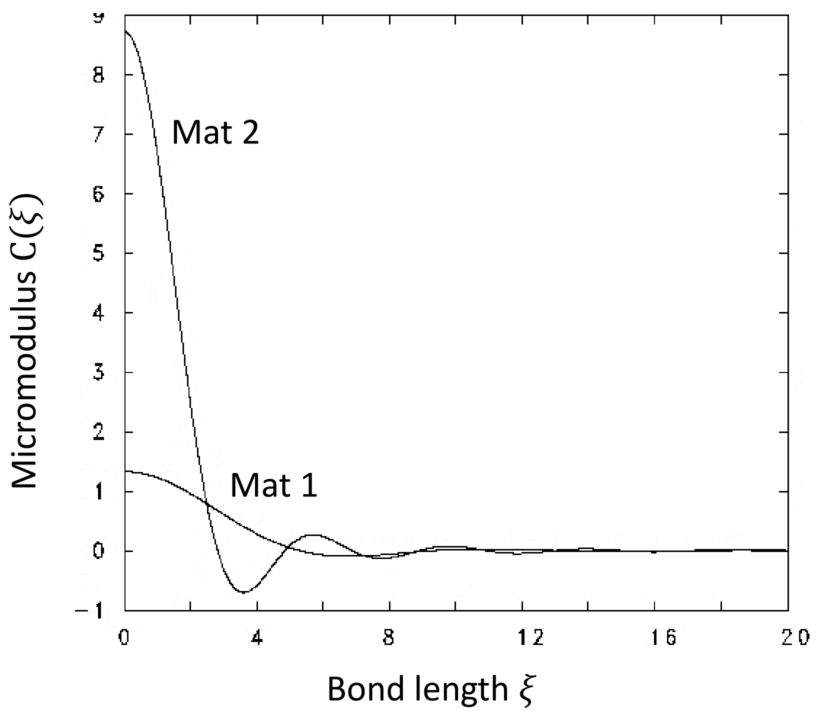


- O. Weckner & S.S., *Int. J. for Multiscale Computational Engineering* (2011).

Higher cutoff frequency leads to narrower micromodulus curve

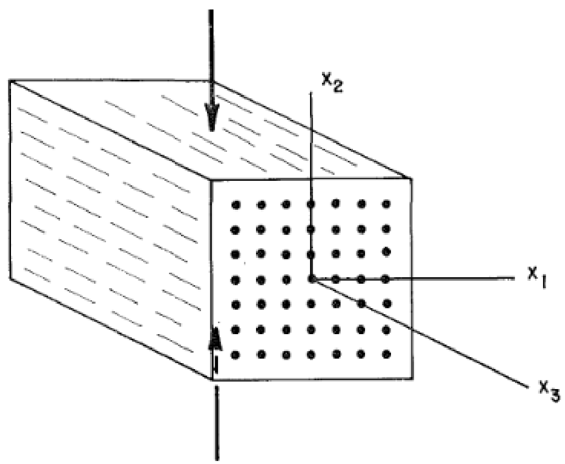
- The limiting case of micromodulus → delta function corresponds to the local theory.

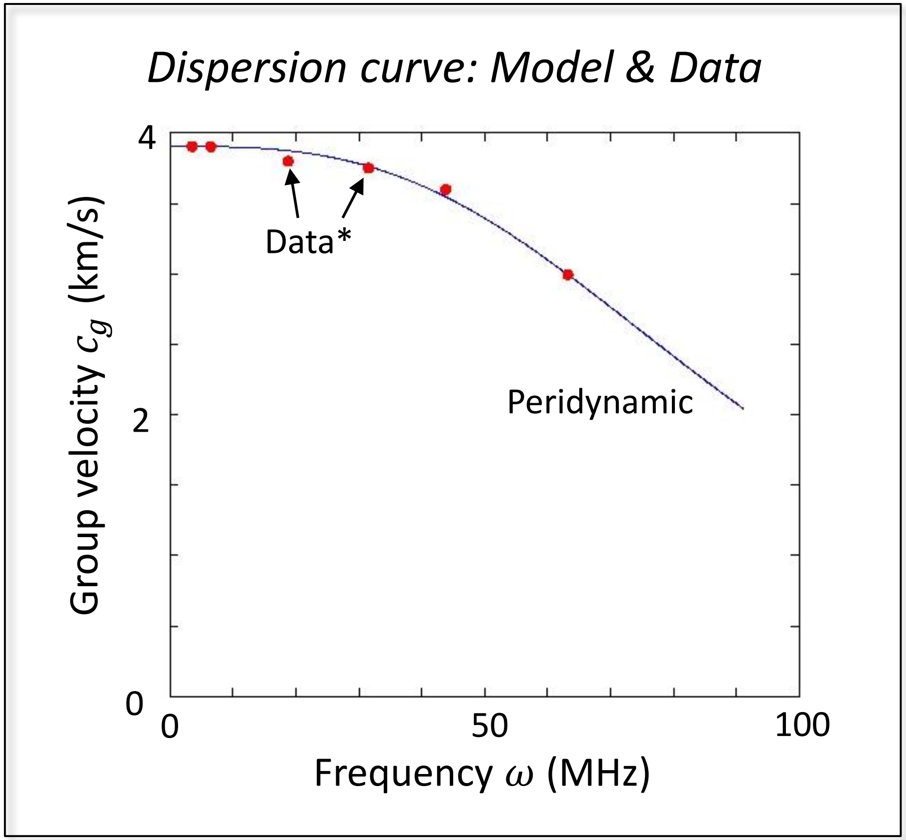




Example: PD model calibrated to a composite dispersion curve

- Boron-epoxy composite.
- Longitudinal waves normal to fibers.
- Compare measured ultrasonic group velocity* with calibrated peridynamic result.





* T. R. Tauchert & A. N. Guzelsu, *J. Applied Mechanics* (1972).

Discussion: Nonlocality in peridynamics

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- Nonlocality emerges from how we choose to model a problem.
- Origins
  - Long-range forces
  - Smoothed degrees of freedom
  - Multiple pathways for flux (of momentum, heat, mass, …)
- Consistency
  - Peridynamics uses a consistently nonlocal approach to the evolution of all fields including damage.