Many validation studies have been done

- First issue of the new *Journal of Peridynamics and Nonlocal Modeling* had a review article by Diehl on published validation to date:

Journal of Peridynamics and Nonlocal Modeling
https://doi.org/10.1007/s42102-018-0004-x

REVIEWS

A Review of Benchmark Experiments for the Validation of Peridynamics Models

Patrick Diehl[1] · Serge Prudhomme[2] · Martin Lévesque[1]

Received: 2 November 2018 / Accepted: 25 December 2018

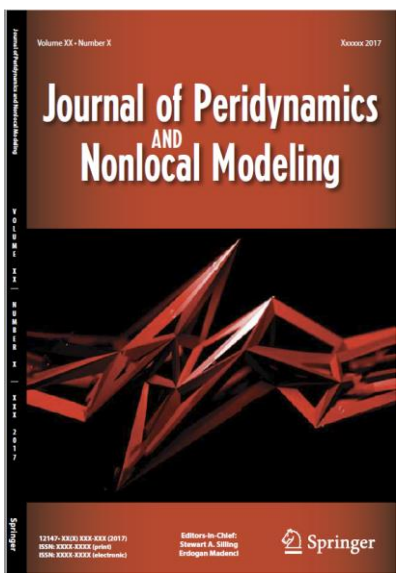


Table 3 Applications of bond-based and state-based peridynamics for the comparison with experimental data

Material	Mechanical test	B	S	Exp	Sim
Composite	Flexural test with an intial crack	✓		[75]	[2]
Composite	Damage growth prediction (six-bolt specimen)	✓		[120]	[96]
Composite	Damage prediction (center-cracked laminates)	✓		[6, 12, 69, 134]	[70]
Composite	Dynamic tension test (prenoteched rectangular plate)	✓		[12, 65]	[58]
Steel	Crack growth (Kalthoff-Winkler)	✓	✓	[66–68]	[3, 52, 114, 144]
Aluminum/Steel	Fracture (compact tension test)	✓		[9, 77, 89, 91]	[135, 141, 142]
Aluminum	Taylor impact test		✓	[4, 21]	[3, 43, 45]
Aluminum (6061-T6)	Ballistic impact test		✓	[132]	[127]
Concrete	Lap-splice experiment	✓		[48]	[48]
Concrete	3-point bending beam	✓	✓	[19, 63]	[7, 51]
Concrete	Failure in a Barazilian disk under compression		✓	[51]	[54]
Concrete	Anchor Bolt Pullout	✓		[128]	[83]
Glass	Dynamic crack propagation (prenotched thin rectangular plate)	✓		[15, 36, 100]	[2, 53, 144]
Glass	Impact damage with a thin polycarbonate backing	✓		[8, 20, 40]	[59]
Glass	Single crack paths (quenched glass plate)	✓		[13, 103, 136]	[71]
Glass	Multiple crack paths (quenched glass plate)	✓		[102, 137]	[71]
Glass	Crack tip propagation speed	✓		[15]	[52, 53, 144]
PMMA	Fast cracks in PMMA	✓		[39]	[2]
PMMA	Tensile test	✓		[124]	[32]
Soda-lime glass	Impact on a two-plate system	✓		[16, 130]	[130]

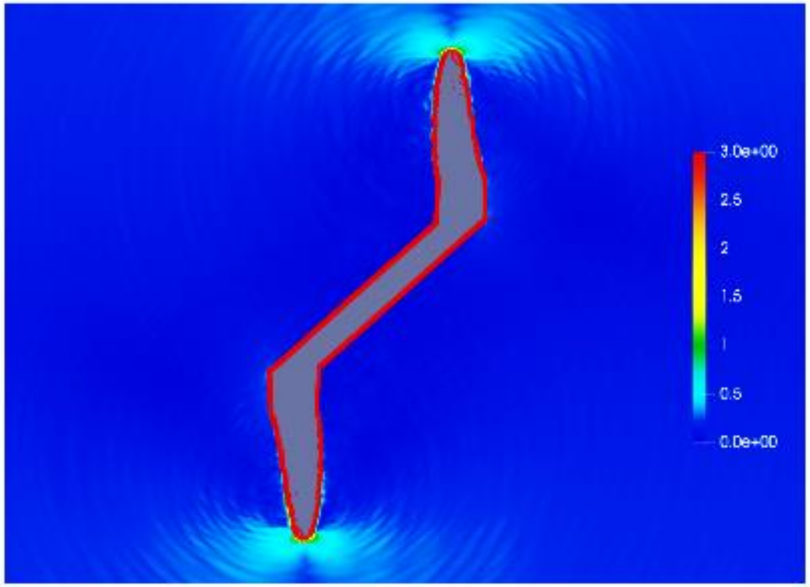
Legend: B refers to bond-based peridynamics, S refers to state-based peridynamics, Exp to experimental data, and Sim to simulation

Sandia National Laboratories

Peridynamics converges as the horizon → 0

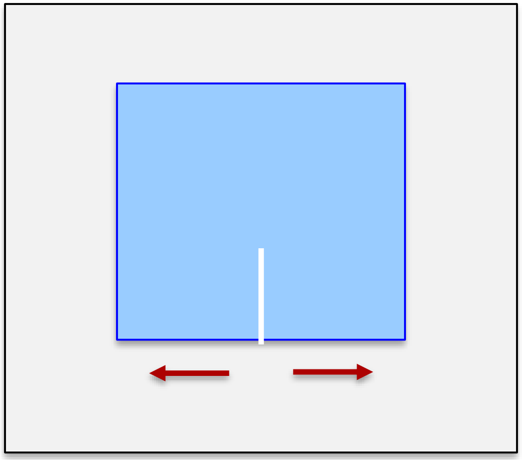
- Linear peridynamics converges to Navier equations of linear elasticity.
- Linear or nonlinear material models converge to a stress-strain relation.
- Problems with nonconvex elastic peridynamic models can converge to nonlinear elasticity with Griffith cracks.

- E. Emmrich & O. Weckner, *Communications in Mathematical Sciences* (2007).
- F. Bobaru et al., Int. *Journal for Numerical Methods in Engineering* (2009).
- T. Mengesha, & Q. Du, *Journal of Elasticity* (2014).
- S.S. & R. B. Lehoucq, *Journal of Elasticity* (2008).
- P. Seleson & D.J. Littlewood, *Computers & Mathematics with Applications* (2016).
- *R. P. Lipton, R. B. Lehoucq, & P.K. Jha, *Journal of Peridynamics and Nonlocal Modeling* (2019).

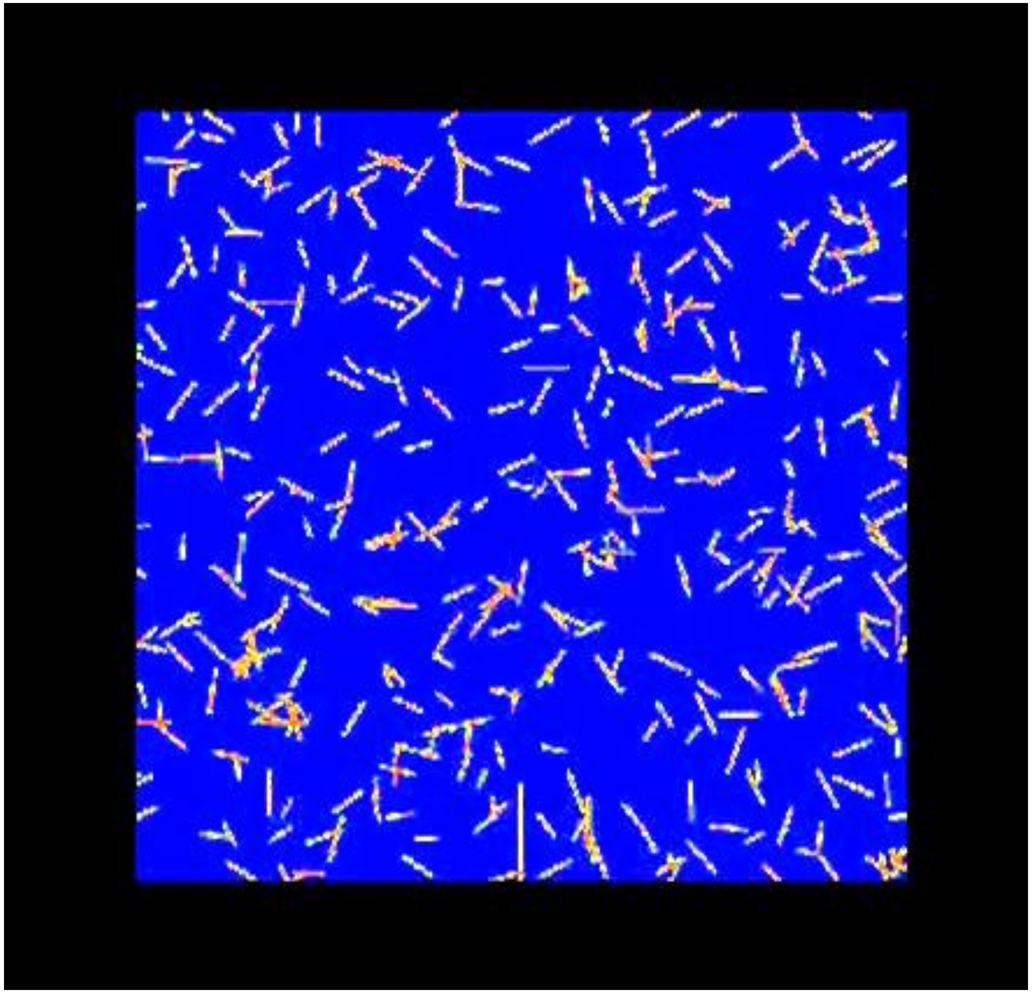


Angled crack growth simulated with a nonconvex peridynamic material model*

Example: Fracture in a brittle plate with a lot of defects

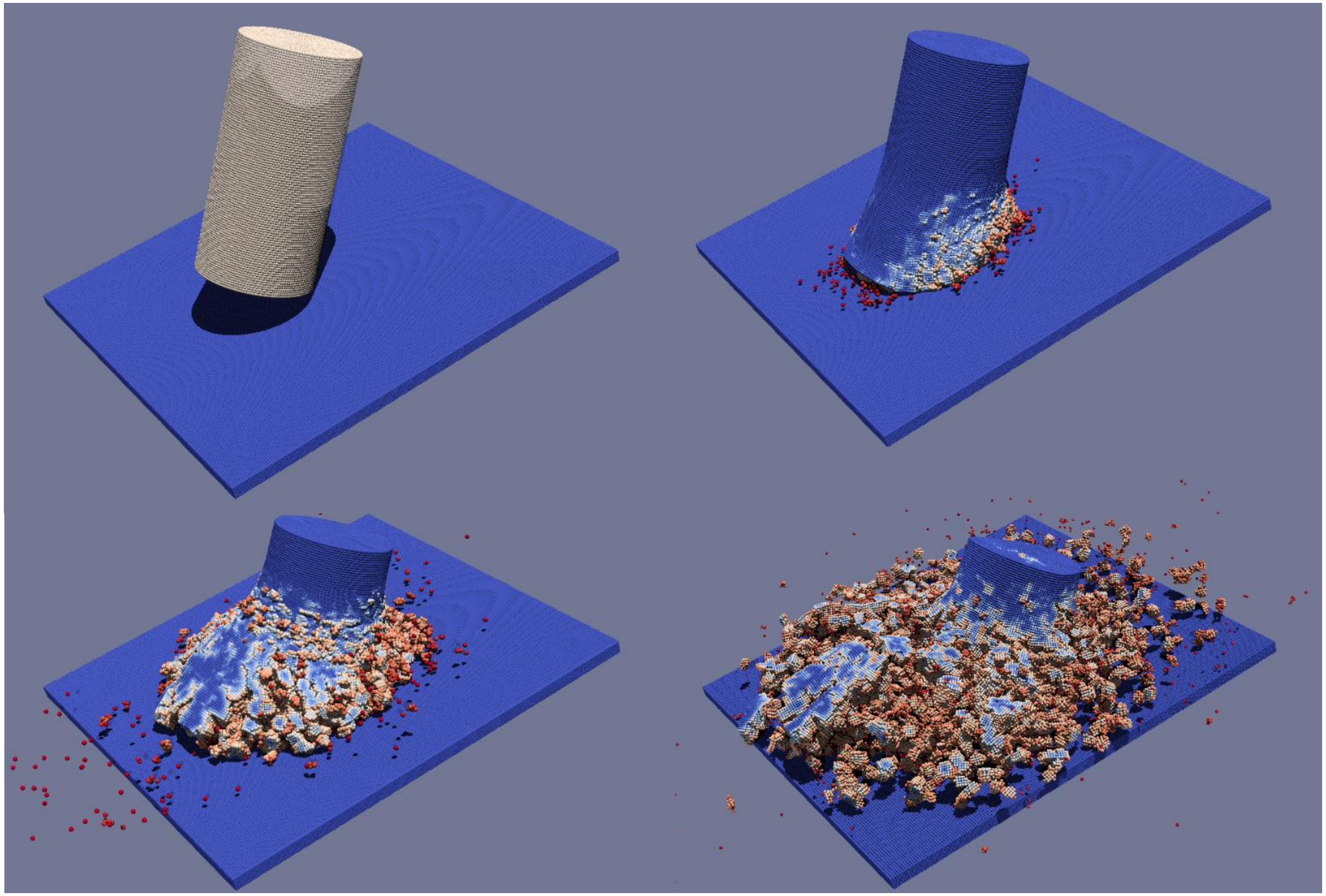


VIDEO



Example: Fragmentation due to impact

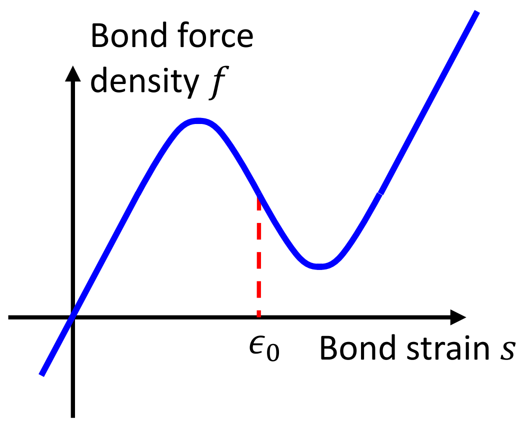
- Brittle cylinder vs. rigid plate at 1km/s.

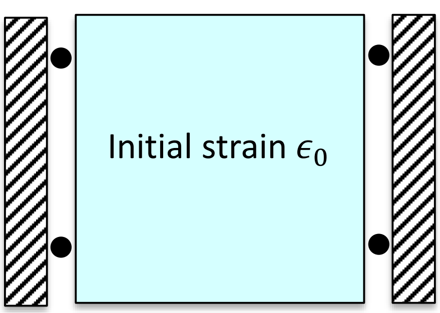


Colors show damage

Example: Microstructure evolution

- Plate with ends fixed. Global strain ϵ_0 is in the unstable part of the material model.
- Complex microstructure appears at first, then simplifies.
- Driving force is the energy stuck in a phase boundary.



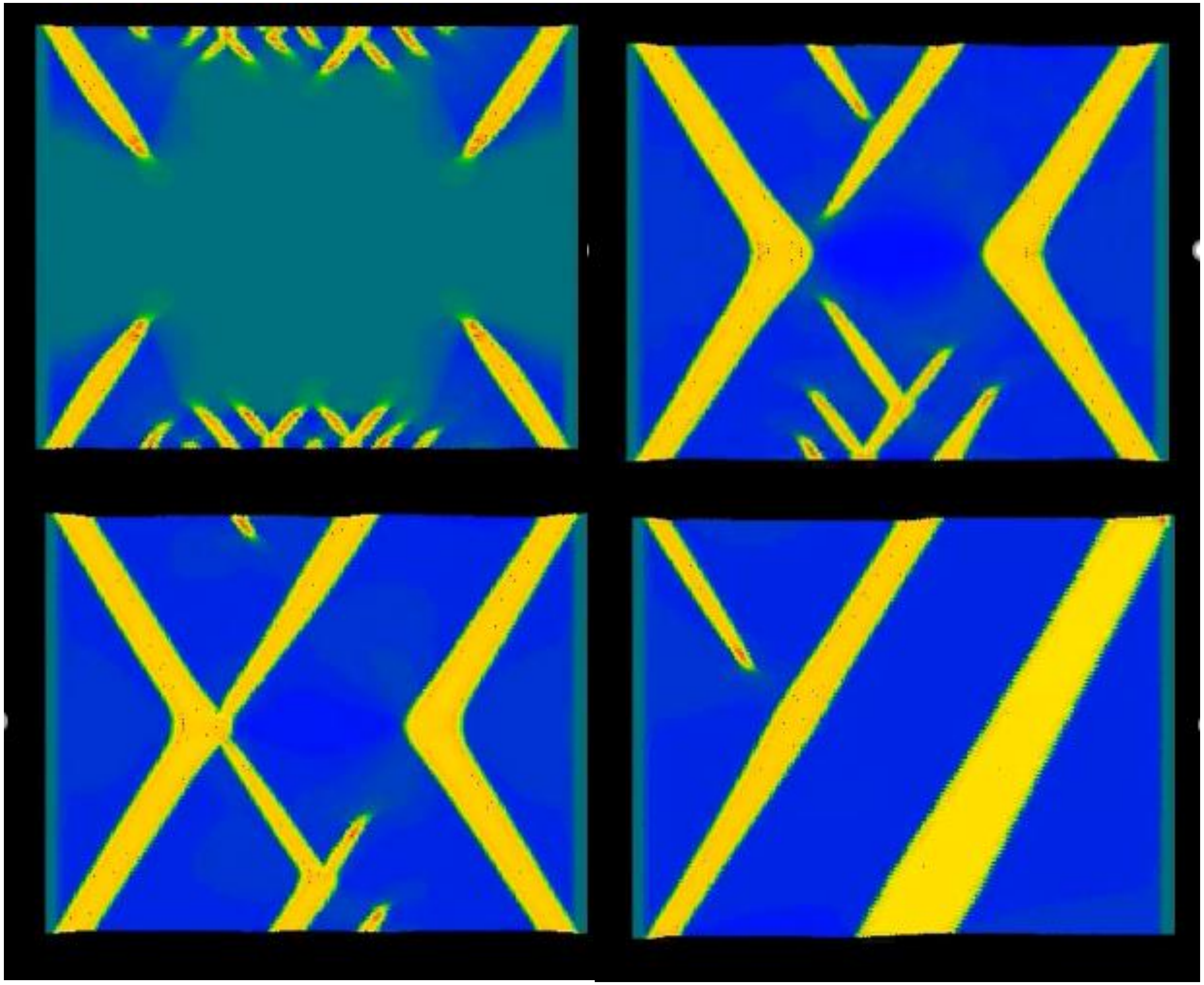


VIDEO

Colors show bond strain

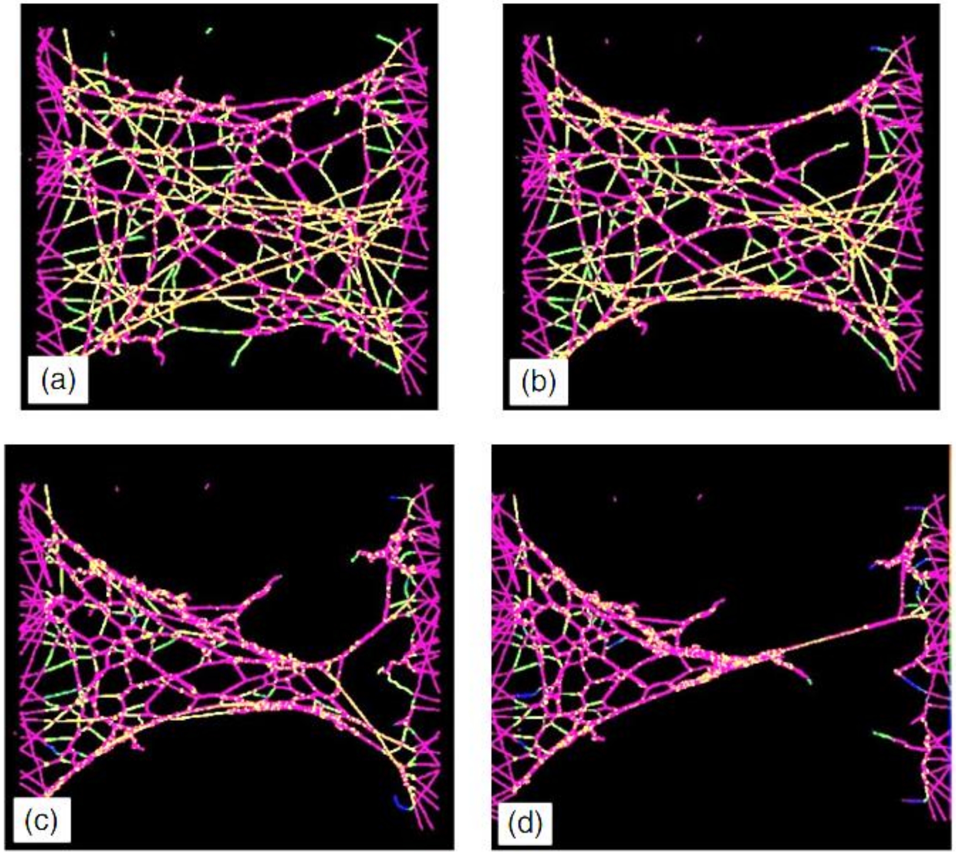
Example: Microstructure evolution

Colors show bond strain



Straightforward case for nonlocality: When there really are long-range forces

- Fracture of nanofiber network held together by Van der Waals forces.



F. Bobaru, *Modelling and Simulation in Materials Science and Engineering* 15, no. 5 (2007): 397.

Smoothing the smallest scale degrees of freedom results in nonlocality

- Try to approximate known, small-scale response (e.g. molecular motion) by a continuous variable, yet retain realistic behavior.
- How to make the connection?
- One approach: Smooth out the small-scale degrees of freedom.
- Example:
  - Heterogeneous infinite bar.

- Small-scale model (local):

$$\rho(x)\ddot{u}(x,t) = \sigma'(x,t) + b(x,t)$$

where ρ=density, u=displacement, σ=stress, and b=body force density.

- Material model:

$$\sigma(x,t) = E(x)u'(x,t)$$

where E=Young's modulus.

Define a smoothed displacement field

- Let $w(z)$ be a smoothing function on $z \in [-\epsilon, \epsilon]$, $\int w = 1$, $w(-z) = w(z)$.

- Define the smoothed displacement field $\bar{u}$ by

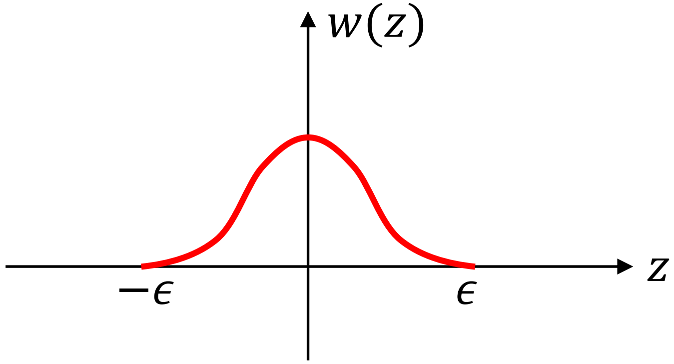
$$\bar{u}(x,t) = \frac{1}{\bar{\rho}(x)} \int_{-\infty}^{\infty} w(p-x)\rho(p)u(p,t)\, dp, \qquad \bar{\rho}(x) := \int_{-\infty}^{\infty} w(p-x)\rho(p)\, dp$$

- Recall

$$\rho(x)\ddot{u}(x,t) = \sigma'(x,t) + b(x,t).$$

- Multiply through by w and integrate, find that

$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = \int_{-\infty}^{\infty} w(x-p)\sigma'(p,t)\, dp + \bar{b}(x,t), \qquad \bar{b}(x,t) := \int_{-\infty}^{\infty} w(x-p)b(p,t)\, dp$$



Evolution equation for smoothed DOFs

Sandia National Laboratories

- Recall

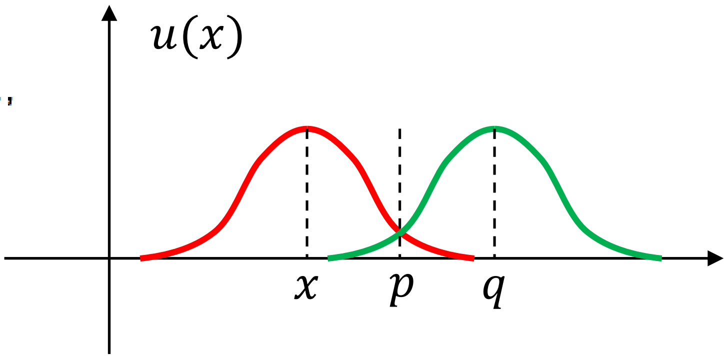
$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = \int_{-\infty}^{\infty} w(x-p)\sigma'(p,t)\,dp + \bar{b}(x,t).$$

- Integrate by parts (surprise!):

$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = -\int_{-\infty}^{\infty} w'(x-p)\sigma(p,t)\,dp + \bar{b}(x,t).$$

- Starting to look nonlocal.
- Let q be defined so that p is halfway between q and x, i.e.,

$$p = \frac{x+q}{2}.$$



- Then

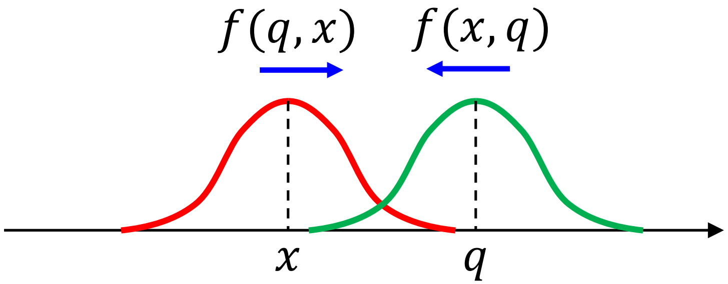
$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = -\frac{1}{2}\int_{-\infty}^{\infty} w'\left(\frac{q-x}{2}\right)\sigma\left(\frac{q+x}{2},t\right)\,dp + \bar{b}(x,t).$$

Evolution equation is nonlocal

- Recall

$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = -\frac{1}{2}\int_{-\infty}^{\infty} w'\left(\frac{q-x}{2}\right)\sigma\left(\frac{q+x}{2},t\right)\,dq + \bar{b}(x,t).$$

- Now define the *pairwise bond force density* by



$$f(q,x) = -\frac{1}{2}w'\left(\frac{q-x}{2}\right)\sigma\left(\frac{q+x}{2},t\right)$$

and define the *horizon* by

$$\delta = 2\epsilon.$$

- We now have

$$\bar{\rho}(x)\ddot{\bar{u}}(x,t) = \int_{x-\delta}^{x+\delta} f(q,x)\,dq + \bar{b}(x,t).$$

- Observe that f has the required symmetry

$$f(x,q) = -f(q,x).$$

Sandia National Laboratories

Need a material model in terms of the smoothed DOFs

- Unfortunately we don't know σ.
- One possibility is to back out u' from the Fourier transform using the convolution theorem:

$$\mathcal{F}\{\bar{u}\} = \mathcal{F}\{w\}\mathcal{F}\{u\} \quad \Longrightarrow \quad u = \mathcal{F}^{-1}\left\{\frac{\mathcal{F}\{\bar{u}\}}{\mathcal{F}\{w\}}\right\}$$

hence

$$\sigma(x) = E(x)\frac{d}{dx}\mathcal{F}^{-1}\left\{\frac{\mathcal{F}\{\bar{u}\}}{\mathcal{F}\{w\}}\right\}.$$

- This is too much work!
- Instead come up with a nonlocal material model.