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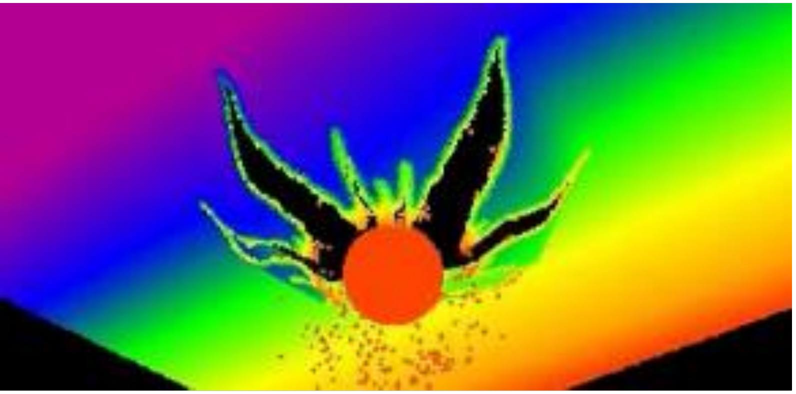

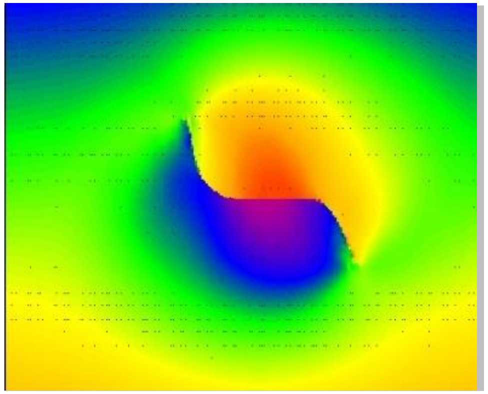

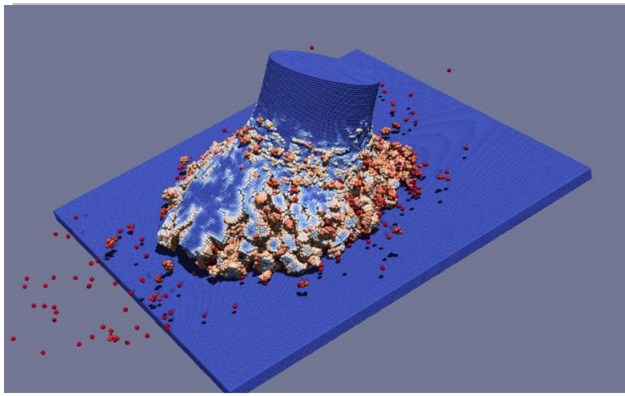

Nonlocality in peridynamics

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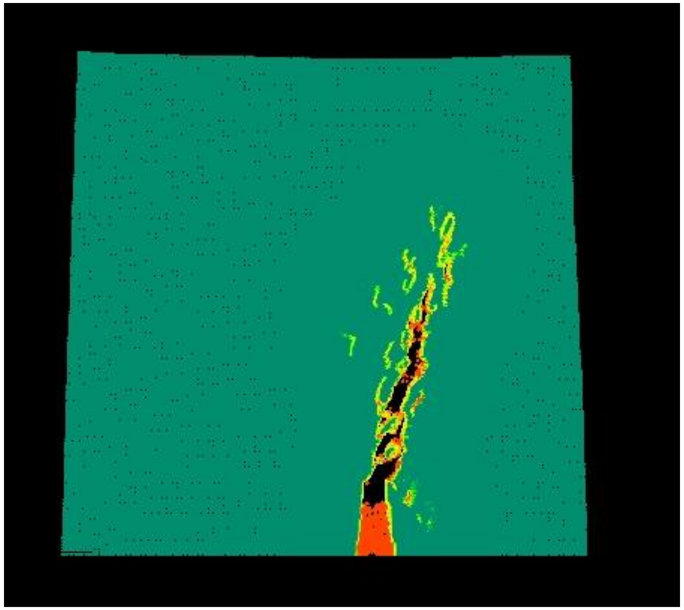
Outline

- Nonlocality
  - It's not as weird as everybody thinks
- Peridynamics background
  - All-in on nonlocality
- Can nonlocality be derived or observed?
  - Long-range forces
  - Smoothed degrees of freedom (homogenization)
  - Multiple pathways for flux
  - Wave dispersion

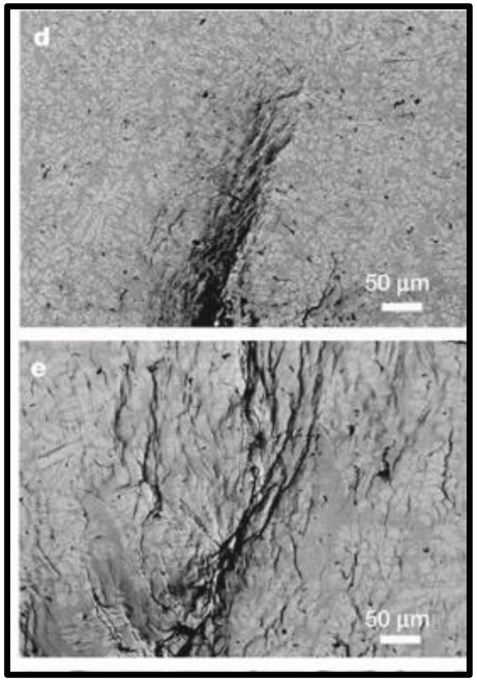
Do we ask too much of the local theory of continuum mechanics?

What peridynamics seeks to accomplish

- Treat material points on or off of evolving discontinuities with the same equations.
- Include long-range forces in the basic equations.
- Fit all this into a thermodynamic framework that's consistent with the mechanics.



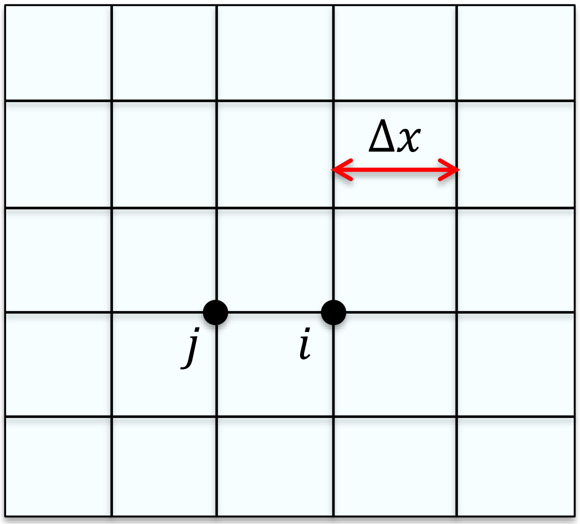
Peridynamic simulation



Metallic glass crack tip*

*Hofmann et al, Nature (2008)

Discretized numerical methods are nonlocal



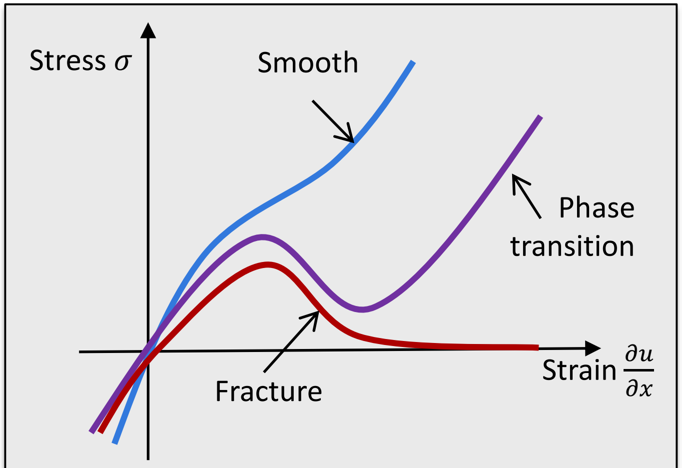
- Node i interacts directly with node j through the finite element equations.
- Interaction is across a finite distance Δx.
- This is a form of nonlocality.
  - Notwithstanding that the result converges to the local result as $\Delta x \to 0$.

Local PDEs get themselves into trouble

- Classical (Cauchy) PDE:

$$\rho\ddot{\mathbf{u}} = \nabla \cdot \boldsymbol{\sigma}\left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\right) + \mathbf{b}.$$

- Many material models $\boldsymbol{\sigma}(\cdot)$ evolve into deformations that are incompatible with the fundamental assumptions.
  - Phase boundaries, shock waves, cracks, ...
- Can't directly treat some important physical effects.
  - Wave dispersion, surface energy, microstructure evolution, long-range forces, ...
- People often take drastic measures if they want to work with this PDE.
  - Element deletion, ...



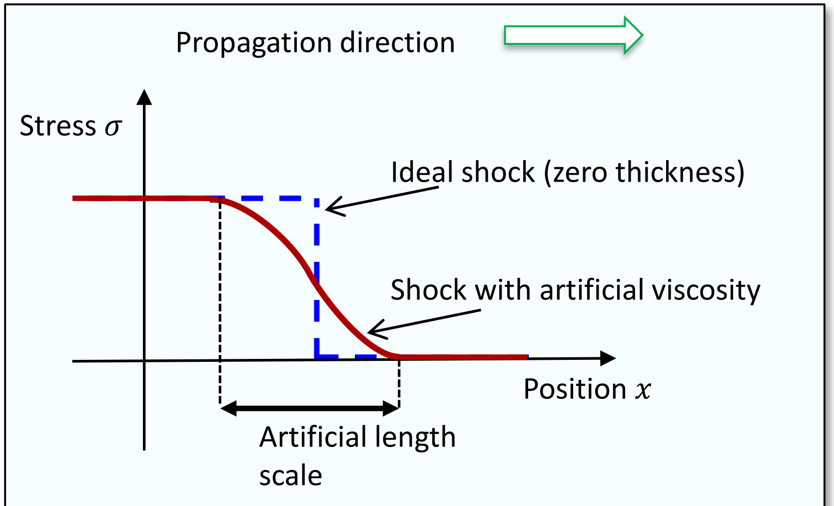
Material models and the features they "try" to predict

These drastic measures often involve nonlocality

- Example: Artificial viscosity spreads out a shock wave and dissipates energy.

$$\rho\ddot{\mathbf{u}} = \nabla \cdot \boldsymbol{\sigma}\left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\right) + \gamma(\nabla \cdot \dot{\mathbf{u}})^2 + \mathbf{b}.$$

- It avoids the need to apply jump conditions across an ideal shock.
- It allows converntional discretization to be used "within" a shock.
- By spreading out a shock it introduces a length scale.
- This is a type of nonlocality.



FEM codes spread out a shock wave over ~6 elements

- J. Von Neumann & R. D. Richtmyer, *J. Appl. Phys*. 21 (1950). 232

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Peridynamics goes all-in on nonlocality

Classification of some theories with respect to local/nonlocality:

PDEs with no length scale:
- Classical continuum mechanics

PDEs with a length scale:
- Micropolar
- Mindlin
- Kroner
- Eringen
- Phase field
- Nonlocal damage
- Plate & shell theories
- Gradient theories

Full nonlocality:
- Kunin
- Peridynamics

- Every fundamental relation in peridynamics is nonlocal in space:
    - Transport
    - Conservation
    - Material models

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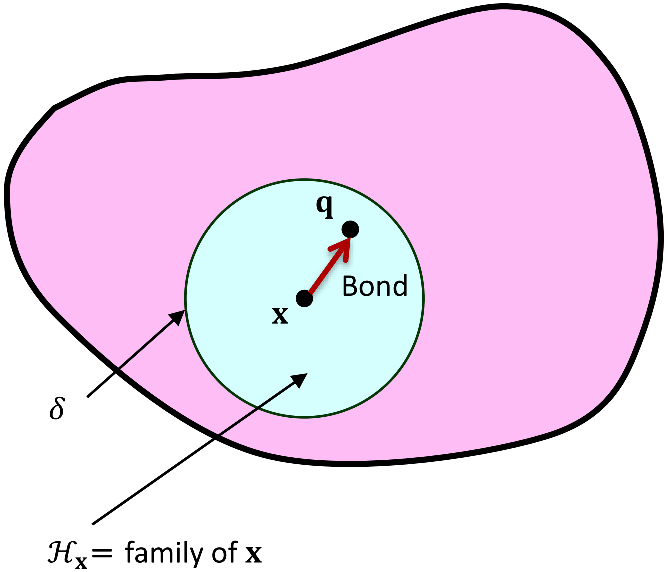
Peridynamic* momentum balance

- Any point $\mathbf{x}$ interacts directly with other points within a distance δ called the "horizon."
- The material within a distance δ of $\mathbf{x}$ is called the "family" of $\mathbf{x}$, $\mathcal{H}_{\mathbf{x}}$.

Peridynamic equilibrium equation

$$\int_{\mathcal{H}_{\mathbf{x}}} \mathbf{f}(\mathbf{q},\mathbf{x})\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x}) = 0$$

f = bond force density (from the material model, which includes damage)



- If $\mathbf{f}$ satisfies $\mathbf{f}(\mathbf{x},\mathbf{q}) = -\mathbf{f}(\mathbf{q},\mathbf{x})$ for all $\mathbf{x}, \mathbf{q}$ then linear momentum is conserved.

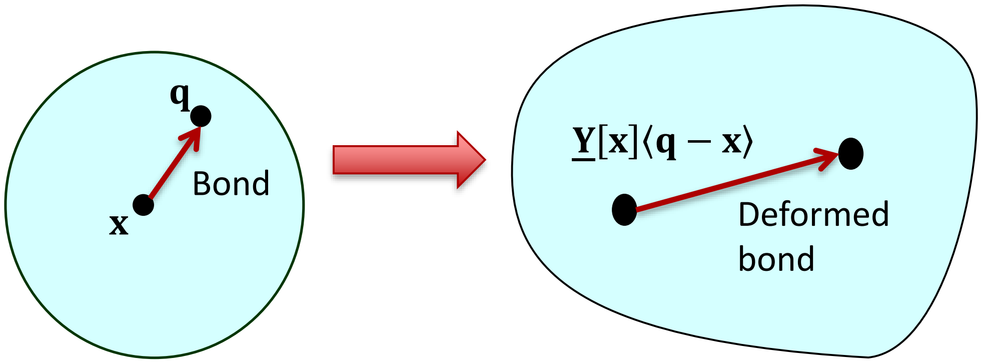
- SS, JMPS (2000)

* Peri (near) + dyne (force)

Formalism for nonlocal interactions: States

- A *state* is a mapping whose domain is all the bonds ξ in a family.

$$\underline{\mathbf{A}}\langle\xi\rangle = \text{something} \qquad \forall \xi \in \mathcal{H}.$$

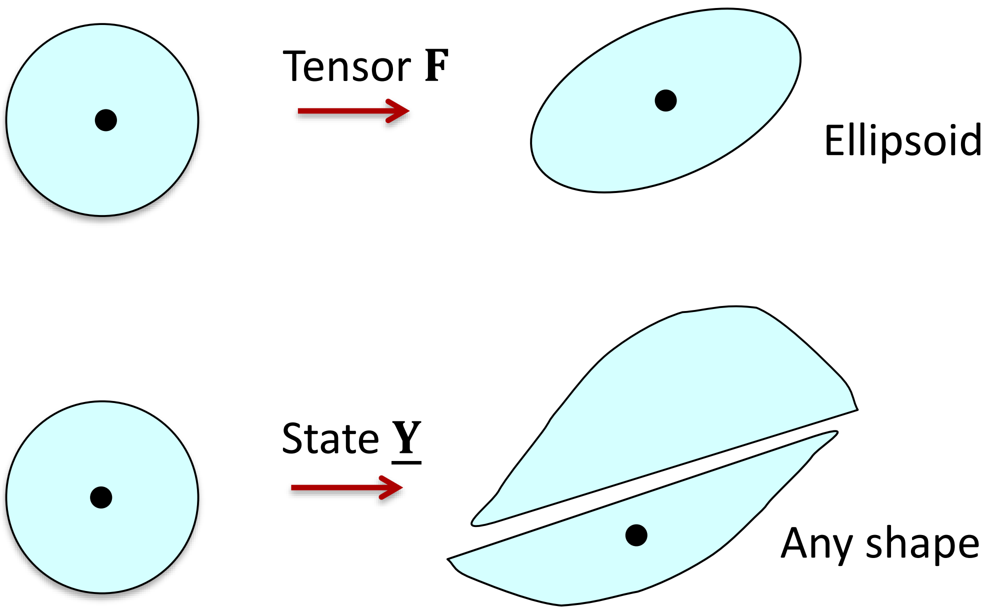


- Deformation state...

$$\underline{\mathbf{Y}}[\mathbf{x}]\langle\mathbf{q}-\mathbf{x}\rangle = \mathbf{y}(\mathbf{q}) - \mathbf{y}(\mathbf{x}) = \text{deformed image of the bond}$$

States: Nonlocal analogues of second order tensors

- Classical theory uses tensors (linear mappings from vectors to vectors).
- Peridynamics uses states (nonlinear mappings from vectors to vectors).



Peridynamic vs. local equations

- Structurally similar but with states instead of local operators.

Relation	*Peridynamic theory*	*Standard theory*
Kinematics	$\underline{\mathbf{Y}}\langle\mathbf{q}-\mathbf{x}\rangle = \mathbf{y}(\mathbf{q}) - \mathbf{y}(\mathbf{x})$	$\mathbf{F}(\mathbf{x}) = \frac{\partial \mathbf{y}}{\partial \mathbf{x}}(\mathbf{x})$
Linear momentum balance	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \int_{\mathcal{H}} \Big(\mathbf{t}(\mathbf{q},\mathbf{x}) - \mathbf{t}(\mathbf{x},\mathbf{q})\Big)\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x})$	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \nabla\cdot\boldsymbol{\sigma}(\mathbf{x}) + \mathbf{b}(\mathbf{x})$
Constitutive model	$\mathbf{t}(\mathbf{q},\mathbf{x}) = \underline{\mathbf{T}}\langle\mathbf{q}-\mathbf{x}\rangle, \qquad \underline{\mathbf{T}} = \hat{\underline{\mathbf{T}}}(\underline{\mathbf{Y}})$	$\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\mathbf{F})$
Angular momentum balance	$\int_{\mathcal{H}} \underline{\mathbf{Y}}\langle\mathbf{q}-\mathbf{x}\rangle \times \underline{\mathbf{T}}\langle\mathbf{q}-\mathbf{x}\rangle\, dV_{\mathbf{q}} = \mathbf{0}$	$\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$
Elasticity	$\underline{\mathbf{T}} = W_{\underline{\mathbf{Y}}}$ (Fréchet derivative)	$\boldsymbol{\sigma} = W_{\mathbf{F}}$ (tensor gradient)
First law	$\dot{\varepsilon} = \underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} + q + r$	$\dot{\varepsilon} = \boldsymbol{\sigma}\cdot\dot{\mathbf{F}} + q + r$

$$\underline{\mathbf{T}} \bullet \dot{\underline{\mathbf{Y}}} := \int_{\mathcal{H}} \underline{\mathbf{T}}\langle\boldsymbol{\xi}\rangle \cdot \dot{\underline{\mathbf{Y}}}\langle\boldsymbol{\xi}\rangle\, dV_{\boldsymbol{\xi}}$$

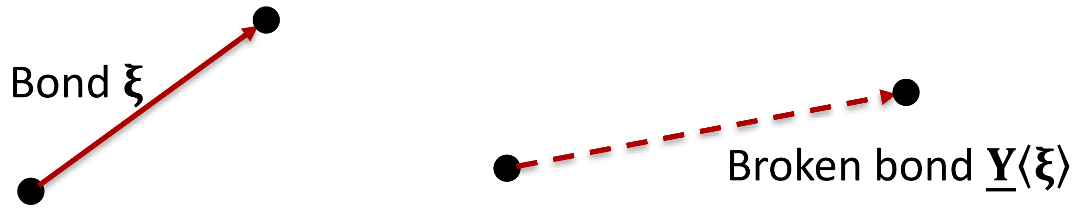
Damage

- Damage is usually treated through *bond breakage*.
- After a bond ξ *breaks* according to some criterion, it no longer carries any force.
- Typical breakage criterion: prescribed *critical bond strain* s_0:

$$s = \frac{|\underline{\mathbf{Y}}\langle\xi\rangle| - |\xi|}{|\xi|} \qquad \text{bond strain.}$$

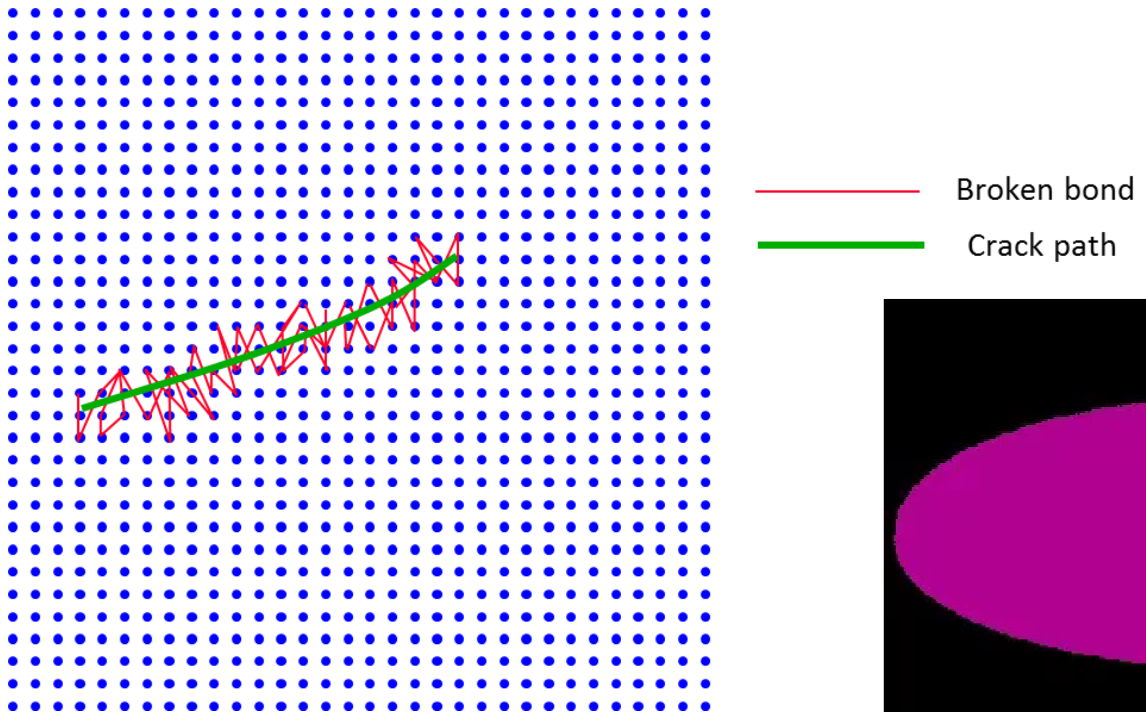
$$s >= s_0 \text{ at some time } t_0$$

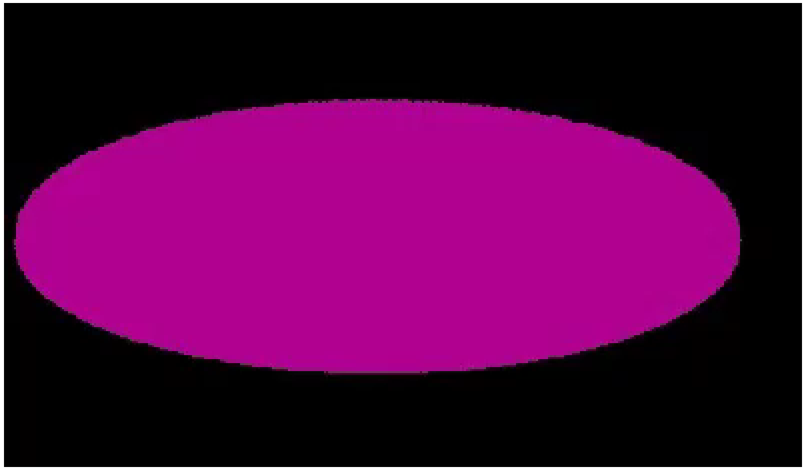
means the bond remains broken for all $t \geq t_0$.



Autonomous crack growth

- Cracks do what they want (grow, arrest, branch, curve, oscillate, …)





- SS & Askari, *Computers and Structures* (2005)