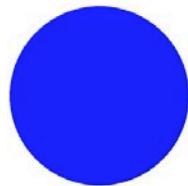


2D Simulation $\sigma_y = 3 \text{ Gpa}$

DB: 2907245.exo
Time:0

Pseudocolor
Var: ep_eq
-2.000
-1.500
-1.000
-0.5000
0.000
Max: 0.000
Min: 0.000

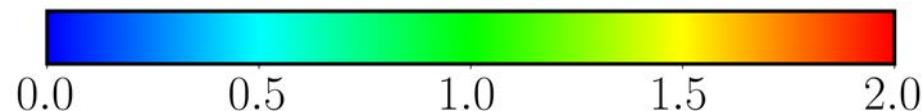
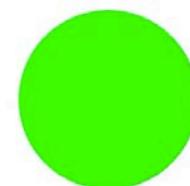
Contour
Var: c
-0.5000
0.000
Max: 0.000
Min: 0.000



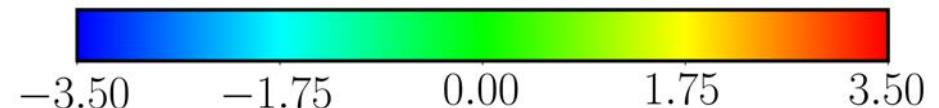
DB: 2907245.exo
Time:0

Pseudocolor
Var: hoop_stress
-3.500
-1.750
0.000
1.750
-3.500
Max: 0.000
Min: 0.000

Contour
Var: c
-0.5000
0.000
Max: 0.000
Min: 0.000

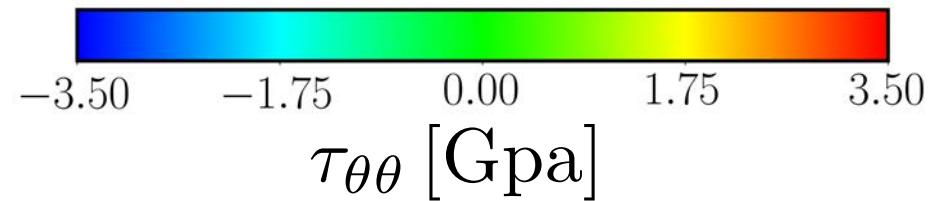
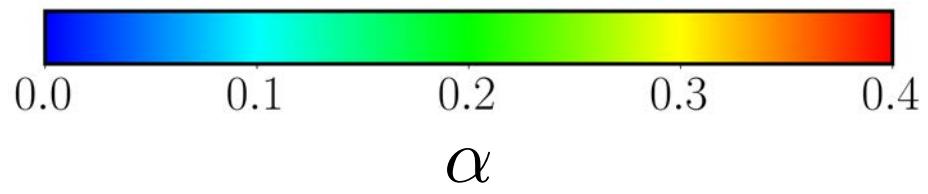
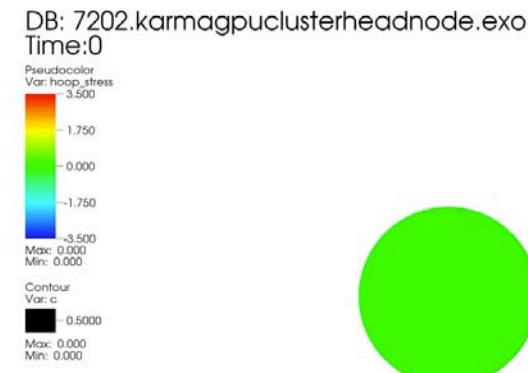
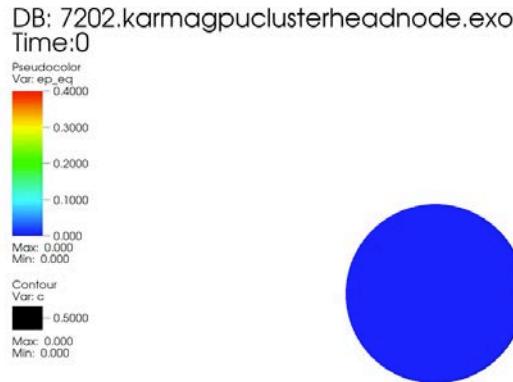


α



$\tau_{\theta\theta} [\text{Gpa}]$

2D Simulation $\sigma_y = 5$ Gpa



Size Effect

Griffith

$$\frac{\tau_c^2}{E} a^n \sim G_c a^{n-1}$$

Pham & Marigo

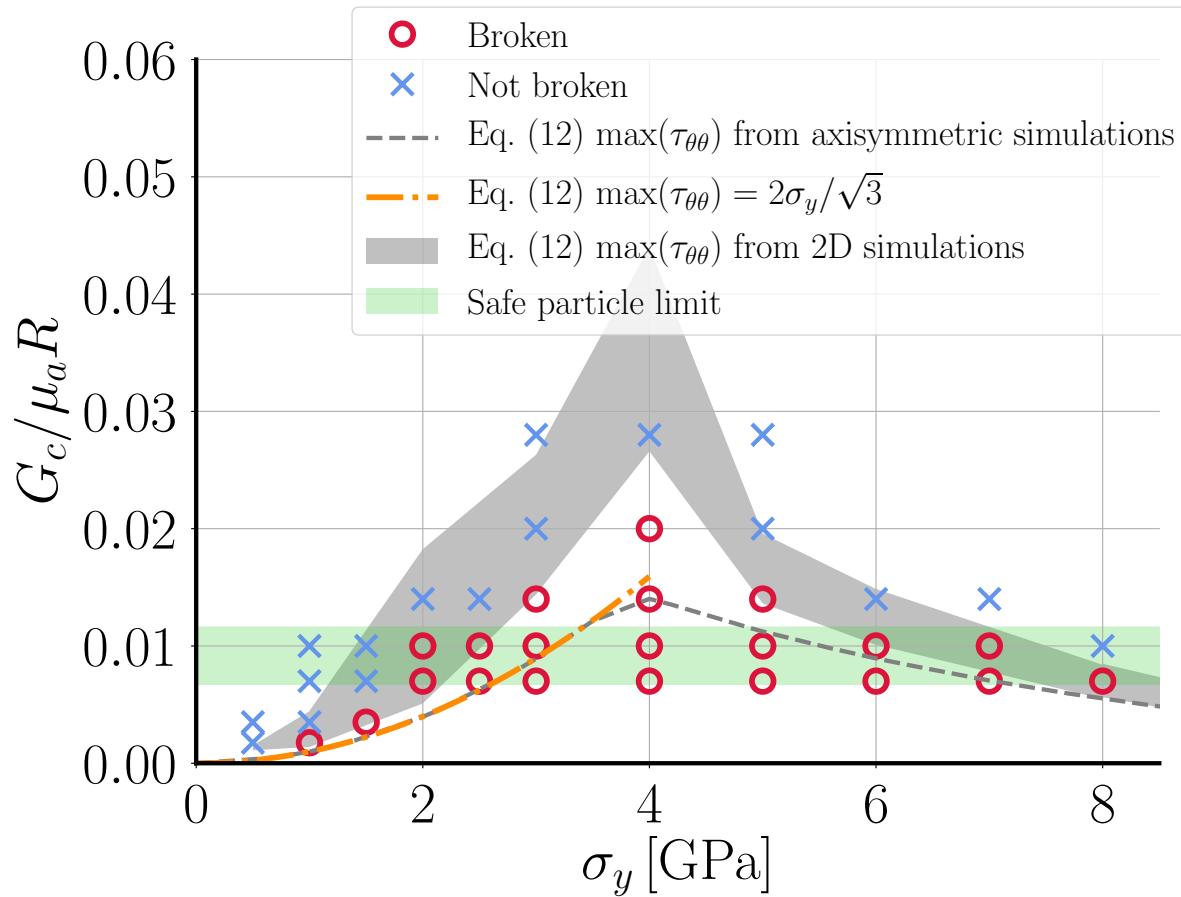
$$\frac{\tau_c^2}{E} \sim \frac{G_c}{\xi}$$

Assumption! $\xi \sim R$

$$\left. \begin{array}{l} \frac{G_c}{\mu_a R} = \mathcal{C} \bar{\xi} (1 - \nu) \left(\frac{\max(\tau_{\theta\theta})}{\mu_a} \right)^2 \end{array} \right\}$$

Pham, K. & Marigo, J.-J. From the onset of damage to rupture: construction of responses with damage localization for a general class of gradient damage models. Continuum Mech. Therm. 1–25 (2013).

Putting it All Together

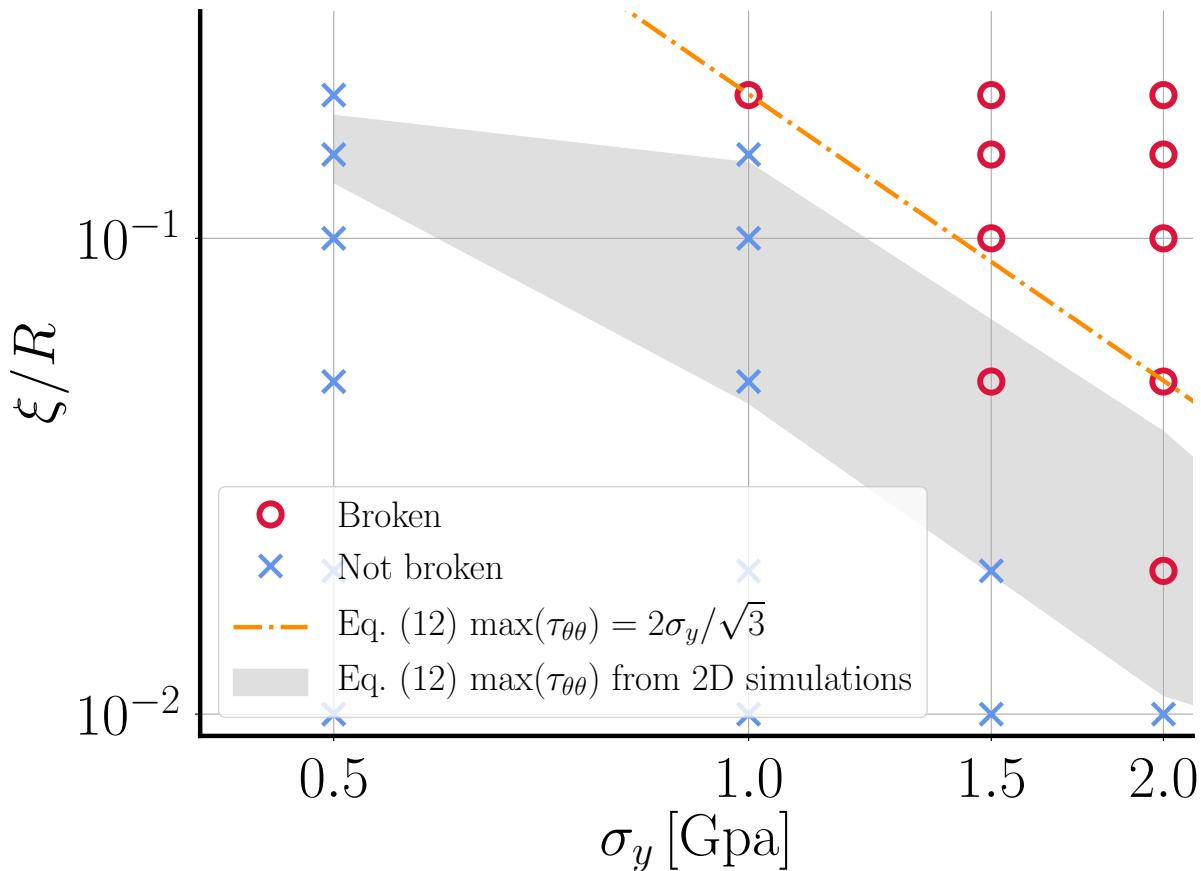


$$\frac{G_c}{\mu_a R} = \mathcal{C}\bar{\xi}(1-\nu) \left(\frac{\max(\tau_{\theta\theta})}{\mu_a} \right)^2$$

Zhao, K. et al. Concurrent reaction and plasticity during initial lithiation of crystalline silicon in lithium-ion batteries. *J. Electrochem. Soc.* 159, A238–A243 (2012).

Zhao, K., Pharr, M., Hartle, L., Vlassak, J. J. & Suo, Z. Fracture and debonding in lithium-ion batteries with electrodes of hollow core–shell nanostructures. *J. Power Sources* 218, 6–14 (2012).

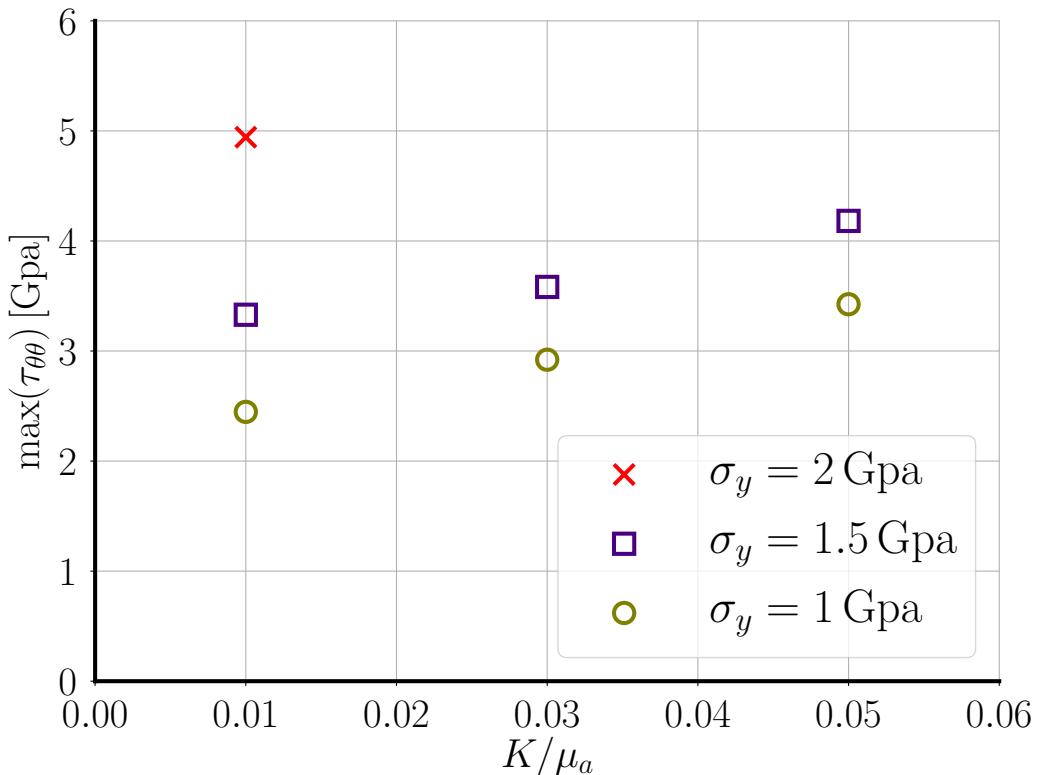
Process Zone Size



$$\frac{G_c}{\mu_a R} = \mathcal{C}\bar{\xi}(1-\nu) \left(\frac{\max(\tau_{\theta\theta})}{\mu_a} \right)^2$$

Zhao, K. et al. Concurrent reaction and plasticity during initial lithiation of crystalline silicon in lithium-ion batteries. *J. Electrochem. Soc.* 159, A238–A243 (2012).
 Zhao, K., Pharr, M., Hartle, L., Vlassak, J. J. & Suo, Z. Fracture and debonding in lithium-ion batteries with electrodes of hollow core–shell nanostructures. *J. Power Sources* 218, 6–14 (2012).

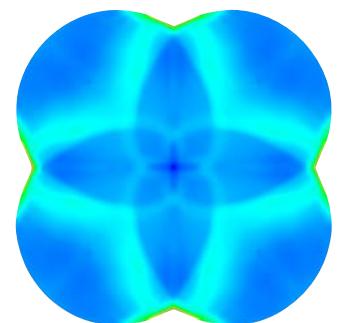
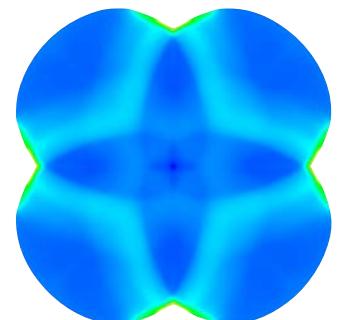
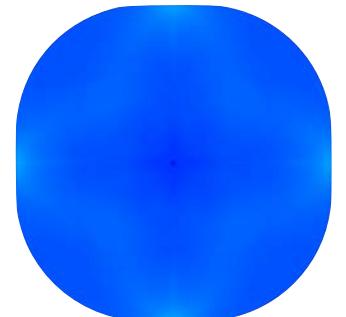
Hardening Effect



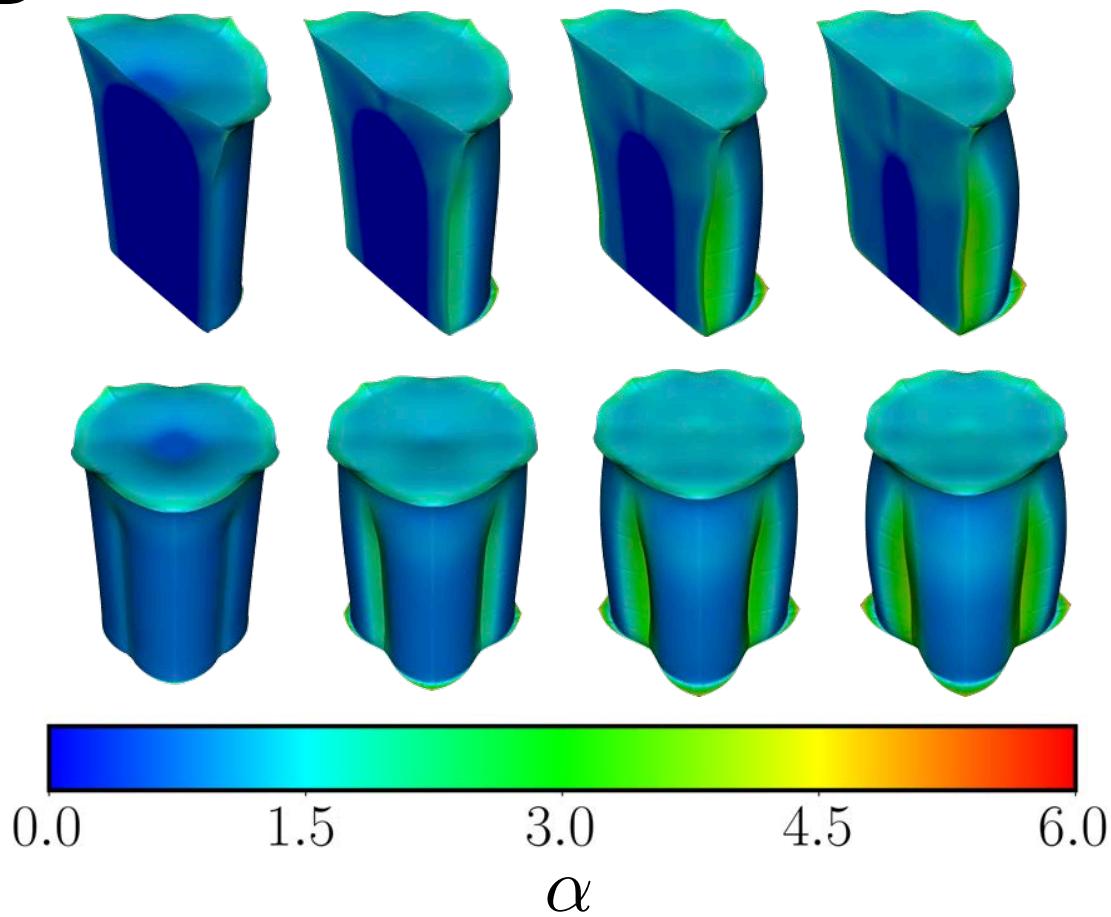
$$K/\mu_a = 1$$

$$K/\mu_a = 0.1$$

$$K/\mu_a = 0.01$$



Going 3D



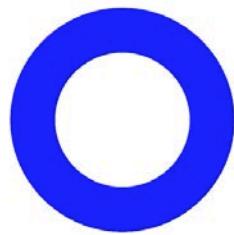
Hollow Cylinder

DB: 3185019.exo

Time:0

Pseudocolor
Var: ep_eq
2.000
1.500
1.000
0.5000
0.000
Min: 0.000

Contour
Var: c
-0.5000
0.000
Min: 0.000

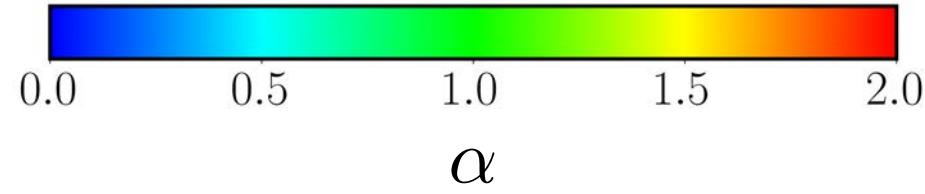
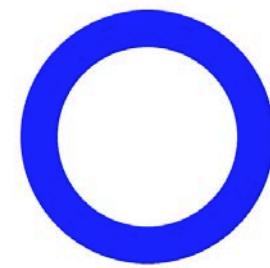


DB: 3248369.exo

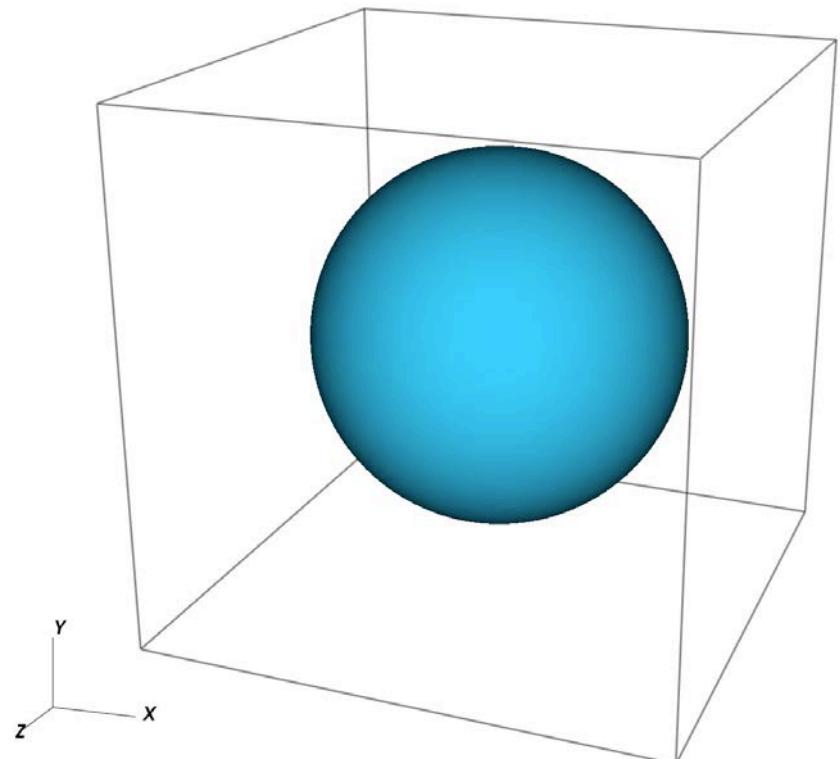
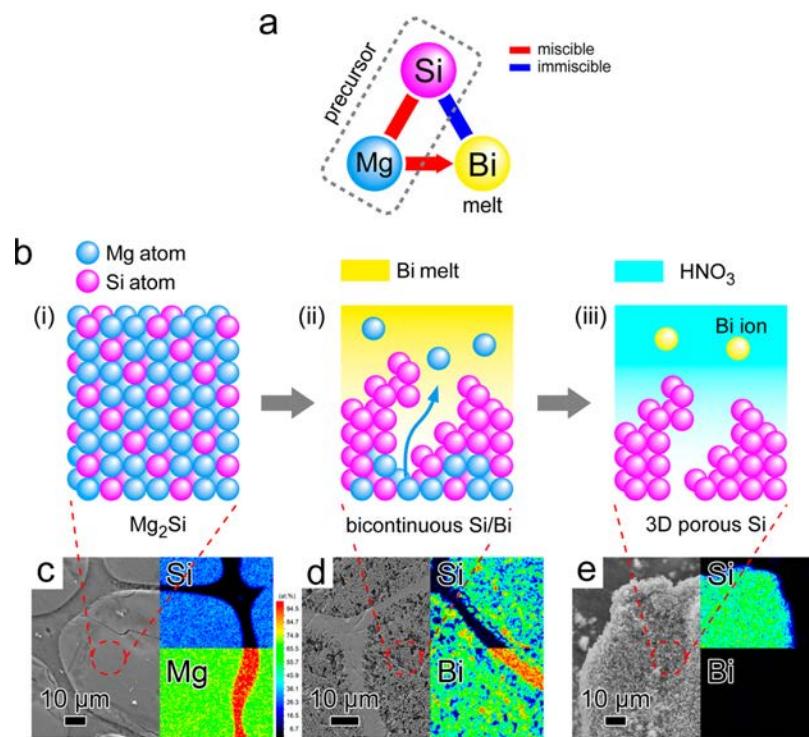
Time:0

Pseudocolor
Var: ep_eq
2.000
1.500
1.000
0.5000
0.000
Min: 0.000

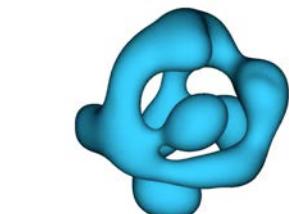
Contour
Var: c
-0.5000
0.000
Min: 0.000



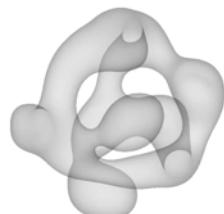
Dealloying of small particles



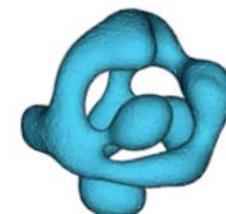
Li insertion in dealloyed geometry

$$\|u\|$$


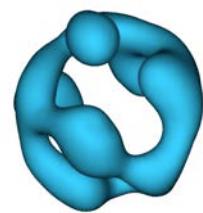
x
y
z

$$\alpha$$


x
y
z

$$\psi$$


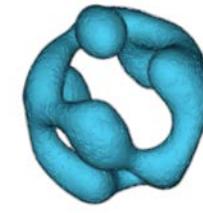
x
y
z



x
y
z



x
y
z



x
y
z

Thank you for your attention.

Open questions:

- Why does J_2 work?
- What does yield stress correspond to?
- What is the plasticity length scale?
- What are effects of plasticity (localization) on initiation?

