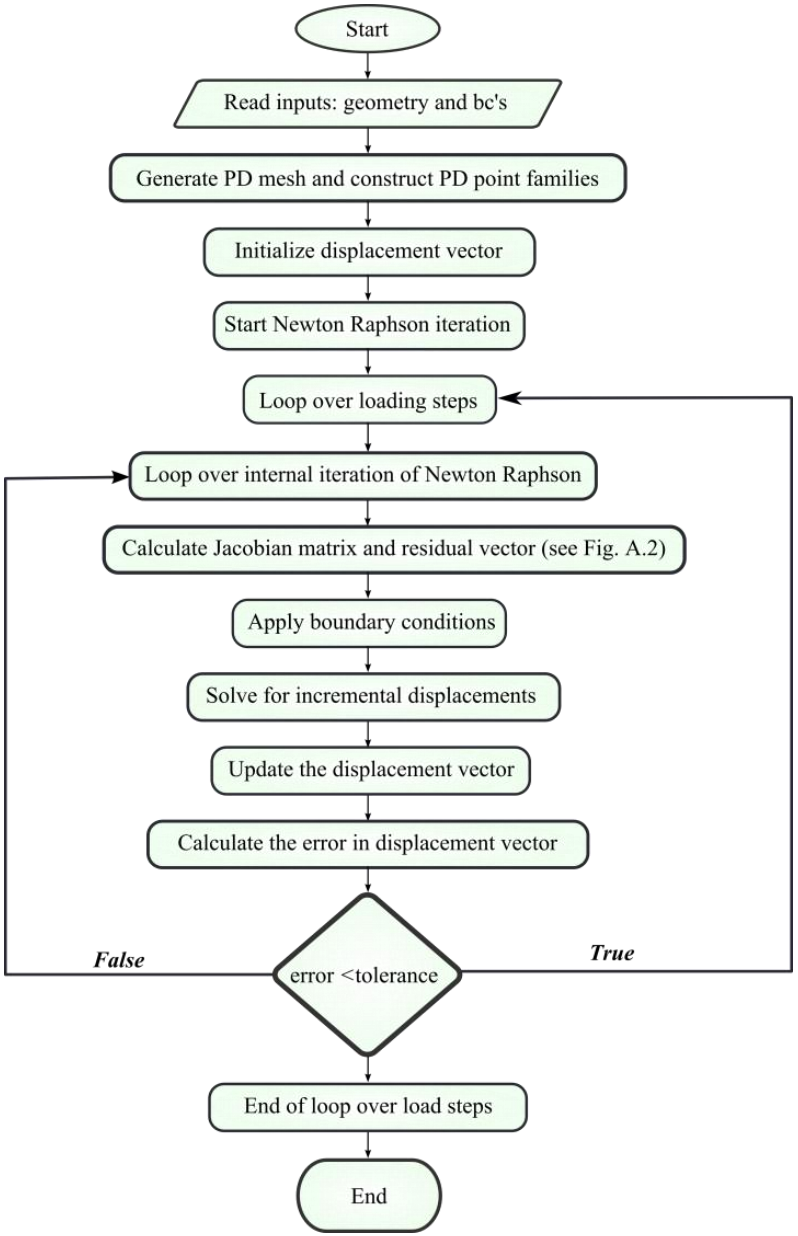
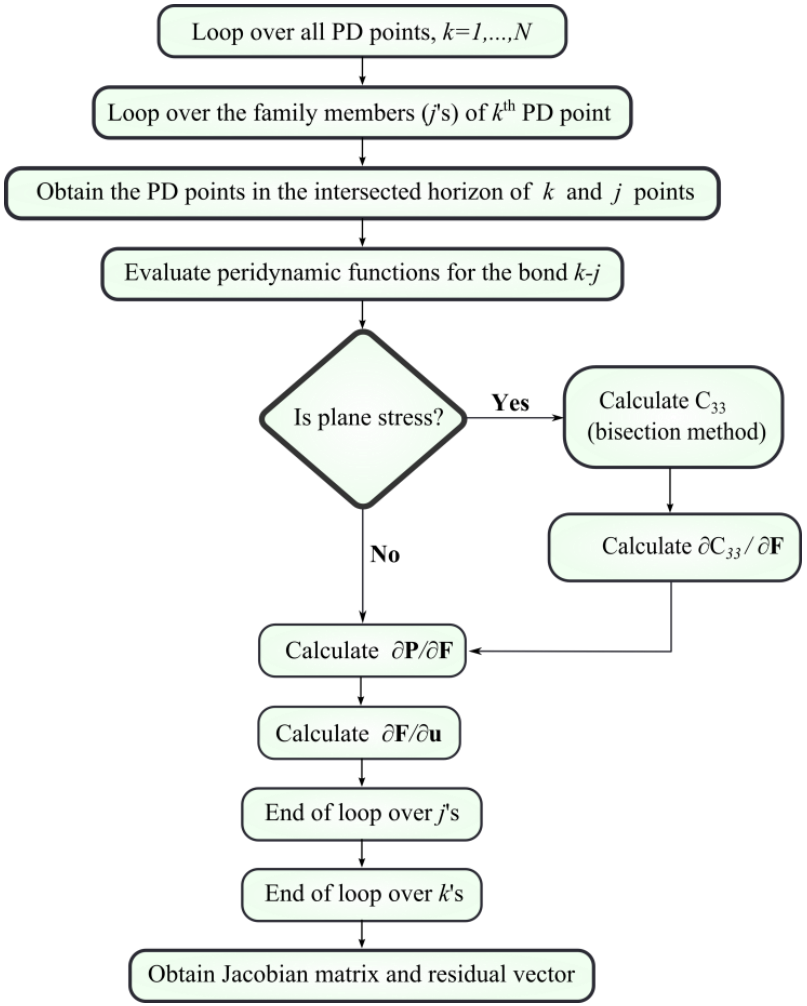
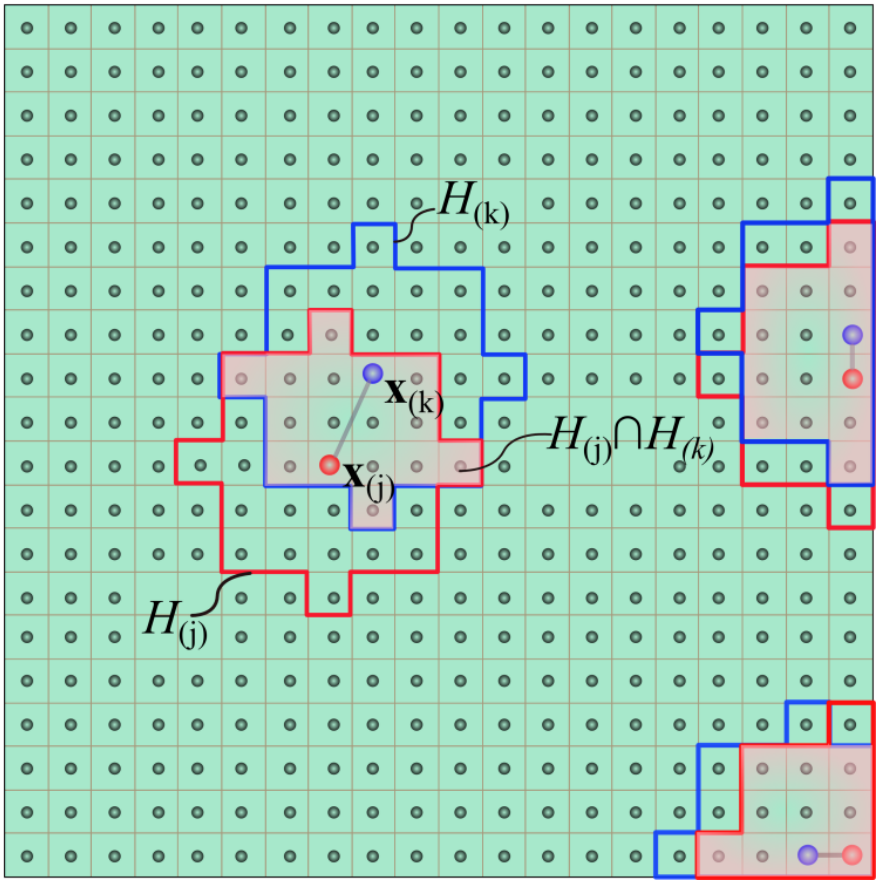
# Flow of calculations



# Discretization and PD family members

**Points within a finite distance (horizon)**



# Failure/damage

**When the bond stretch reaches its critical value, bond breakage occurs**

$$\mu(\mathbf{x}'-\mathbf{x}) = \begin{cases} 1, & s \le s_c \\ 0, & s \ge s_c \end{cases}$$

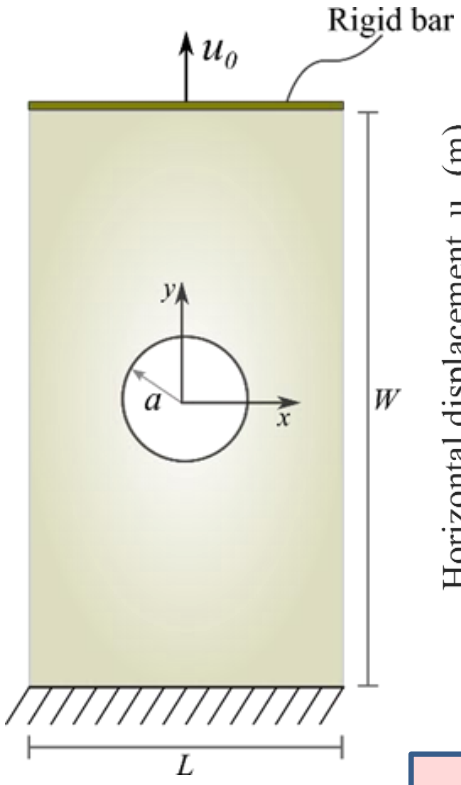
$$s = \frac{|\mathbf{y}'-\mathbf{y}|}{|\mathbf{x}'-\mathbf{x}|}$$

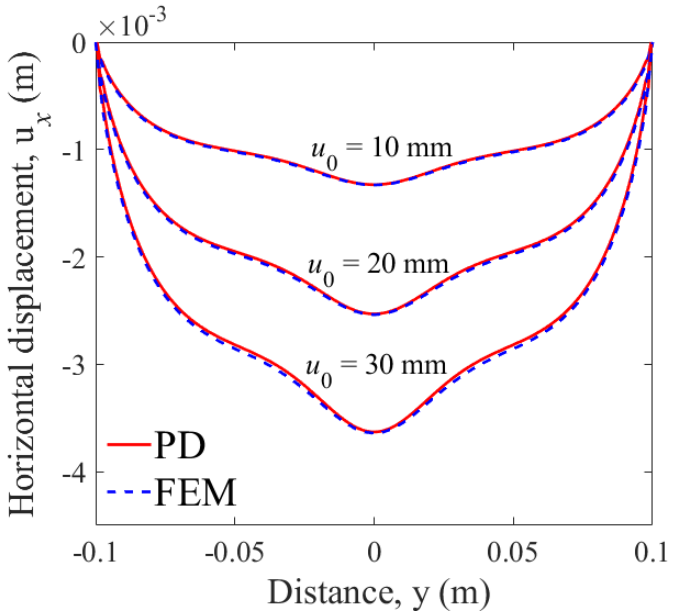
$$\mathbf{L}^{\mathrm{PD}}(\mathbf{x},t) = \int_{H_{\mathbf{x}}} \left( \mu\phi_{\xi}(\mathbf{x},\mathbf{x}')\mathbf{P}_{\xi}\,\mathbf{g}_{\xi}(\mathbf{x}) - \mu\phi_{\xi}(\mathbf{x}',\mathbf{x})\mathbf{P}'_{\xi}\,\mathbf{g}_{\xi}(\mathbf{x}') \right) dV_{\mathbf{x}'}$$

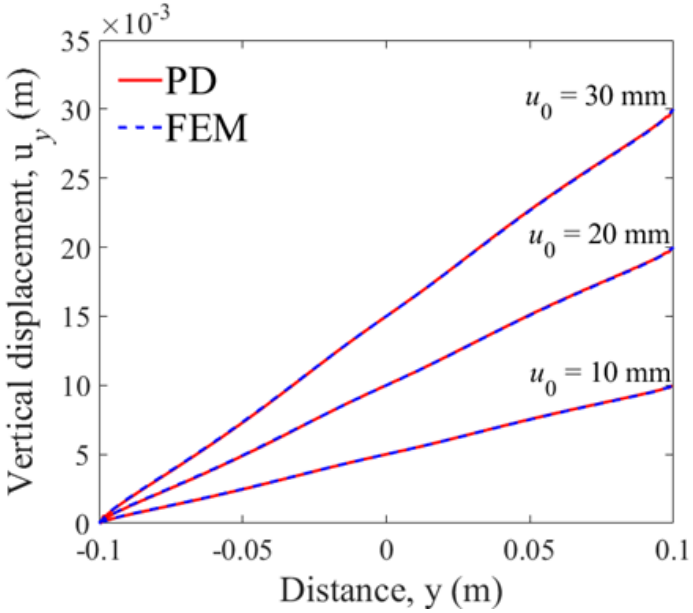
$$\varphi(\mathbf{x}) = 1 - \frac{\int_{H_{\mathbf{x}}} \mu(\mathbf{x}'-\mathbf{x})\,dV_{\mathbf{x}'}}{\int_{H_{\mathbf{x}}} dV_{\mathbf{x}'}}$$

**Local damage is the ratio of number of broken bonds to total number of bonds**

# Rubber sheet with a hole under tension




$L = 80\text{mm}, W = 200\text{mm}, t = 3\text{mm}$

$K = 2.68$ MPa, $\mu = 0.268$ MPa

$\Delta u_0 = 0.25\,\text{mm}$

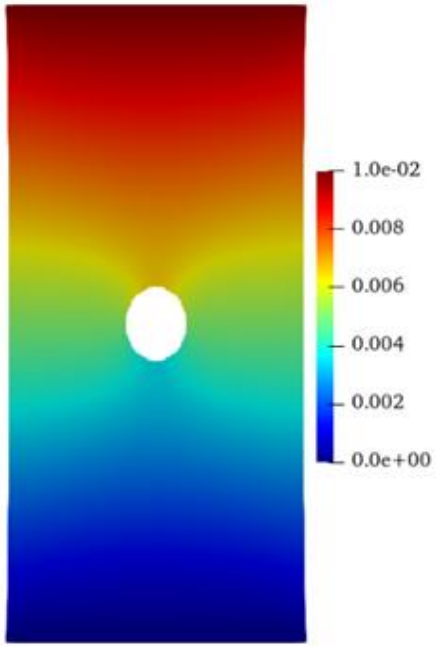
$A_{(k)} = 1.0\ \text{mm}^2$

$\delta = 3.015\Delta x$

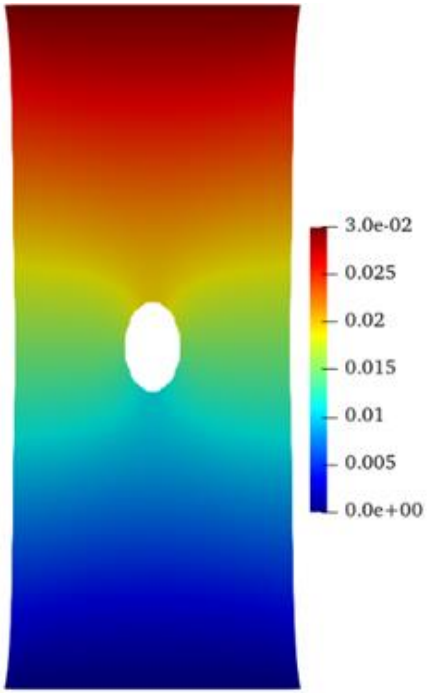
# PD vertical displacements

**Applied stretch**

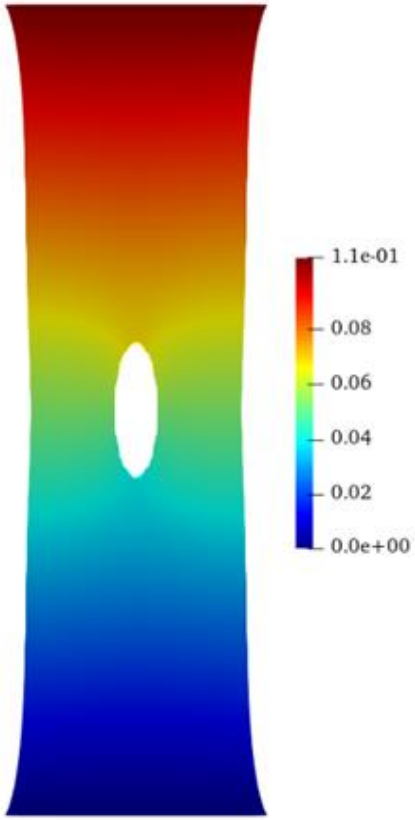
**10 mm**



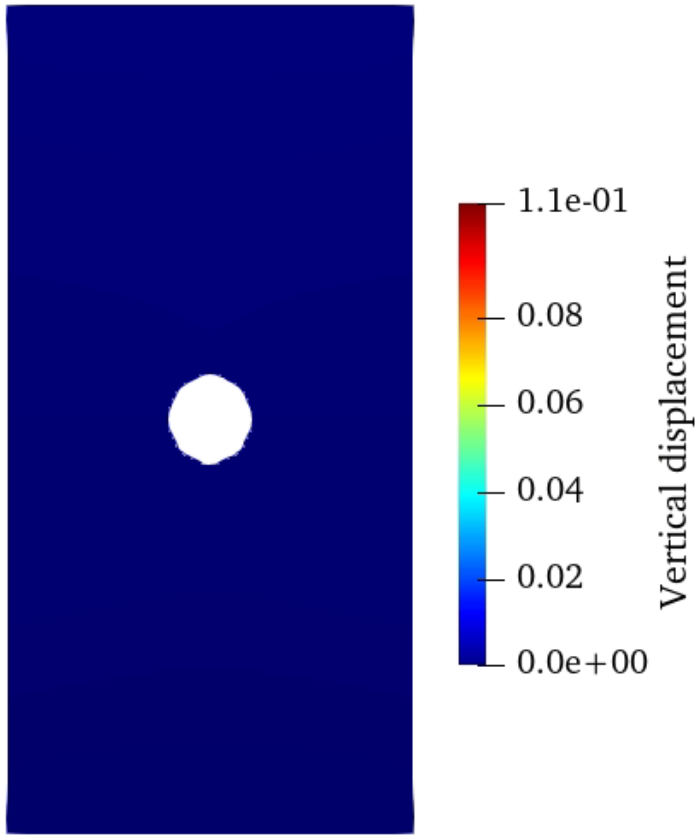

**30 mm**



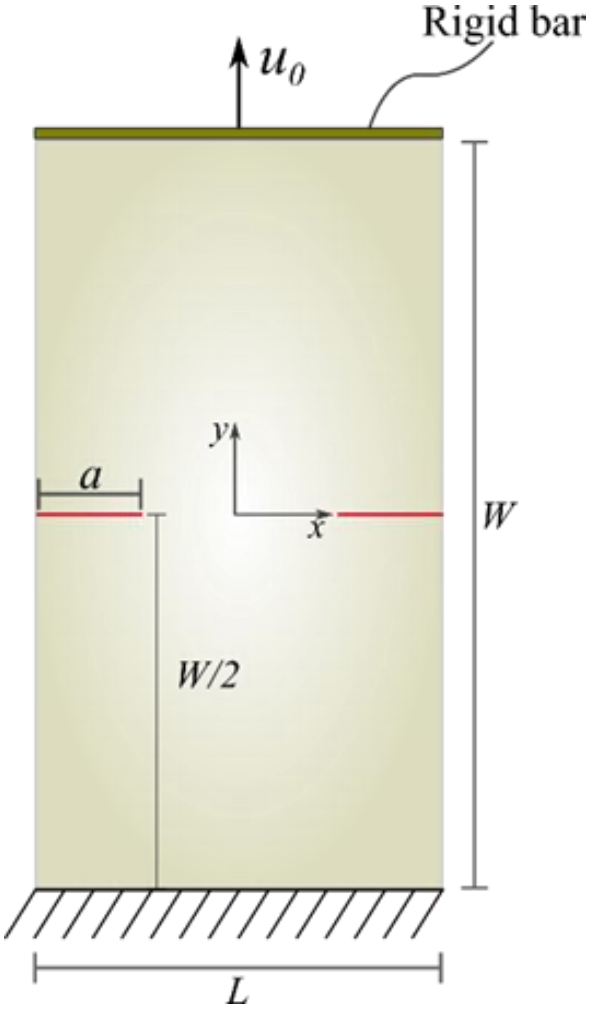

**110 mm**



Movie



# Rubber sheet with double edge notches




$L = 80\text{mm}, W = 200\text{mm}, t = 3\text{mm}$

$a = 12, 16, 20, 24, 28\text{mm}$

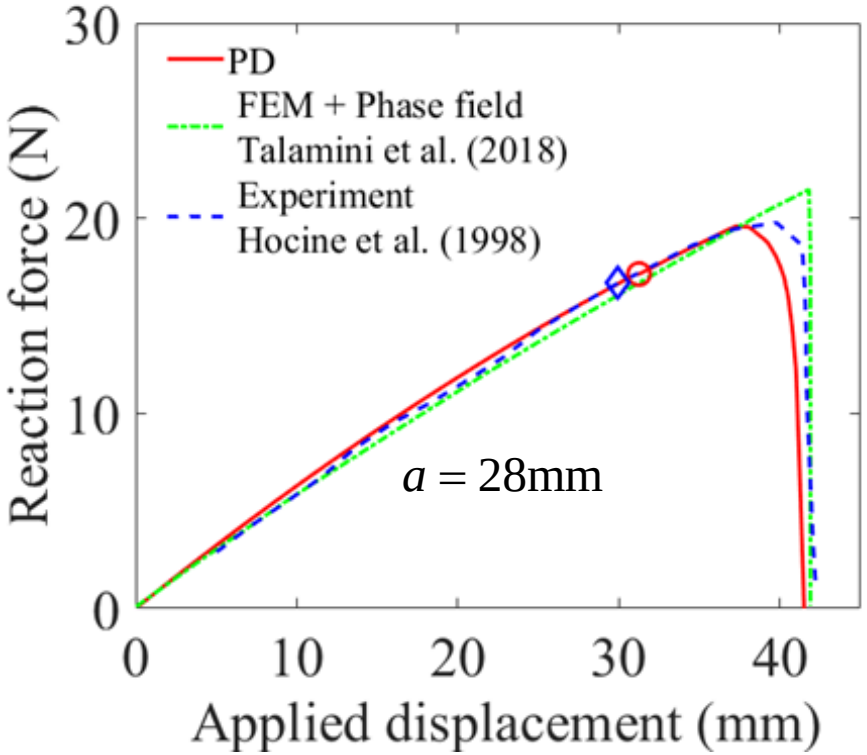
$K = 2.68\ \text{MPa},\ \ \mu = 0.268\ \text{MPa}$

$\Delta u_0 = 0.25\,\text{mm}$

$s_c = 3.1$

$A_{(k)} = 1.0\ \text{mm}^2$

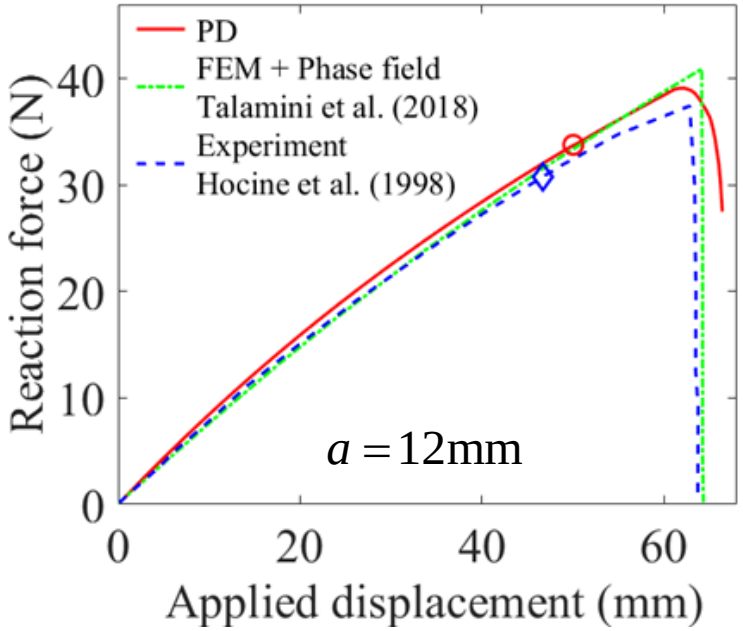
$\delta = 3.015\Delta x$

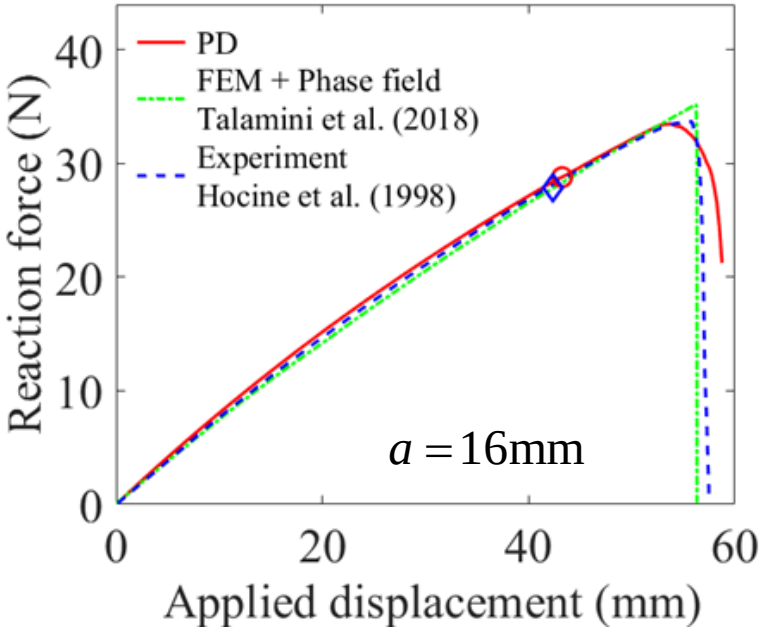


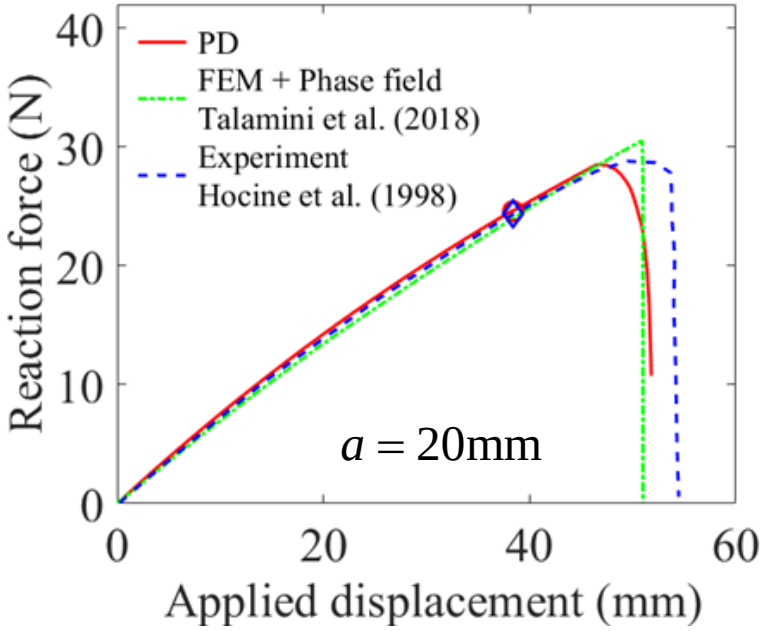


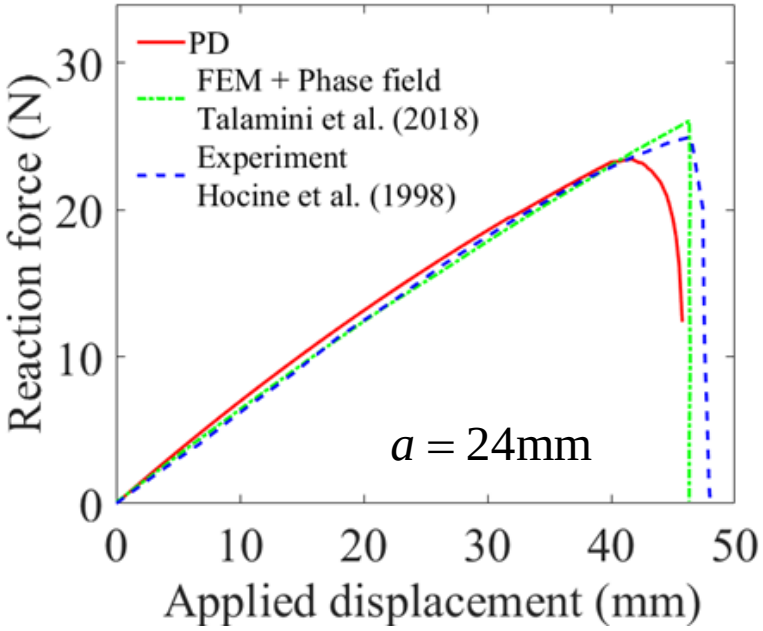
Hocine NA, Abdelaziz MN and Mesmacque G. Experimental and numerical investigation on single specimen methods of determination of J in rubber materials. Int J Fract 1998; 94: 321–338.

B Talamini, Y Mao, L Anand Progressive damage and rupture in polymers Journal of the Mechanics and Physics of Solids 2018 111, 434-457

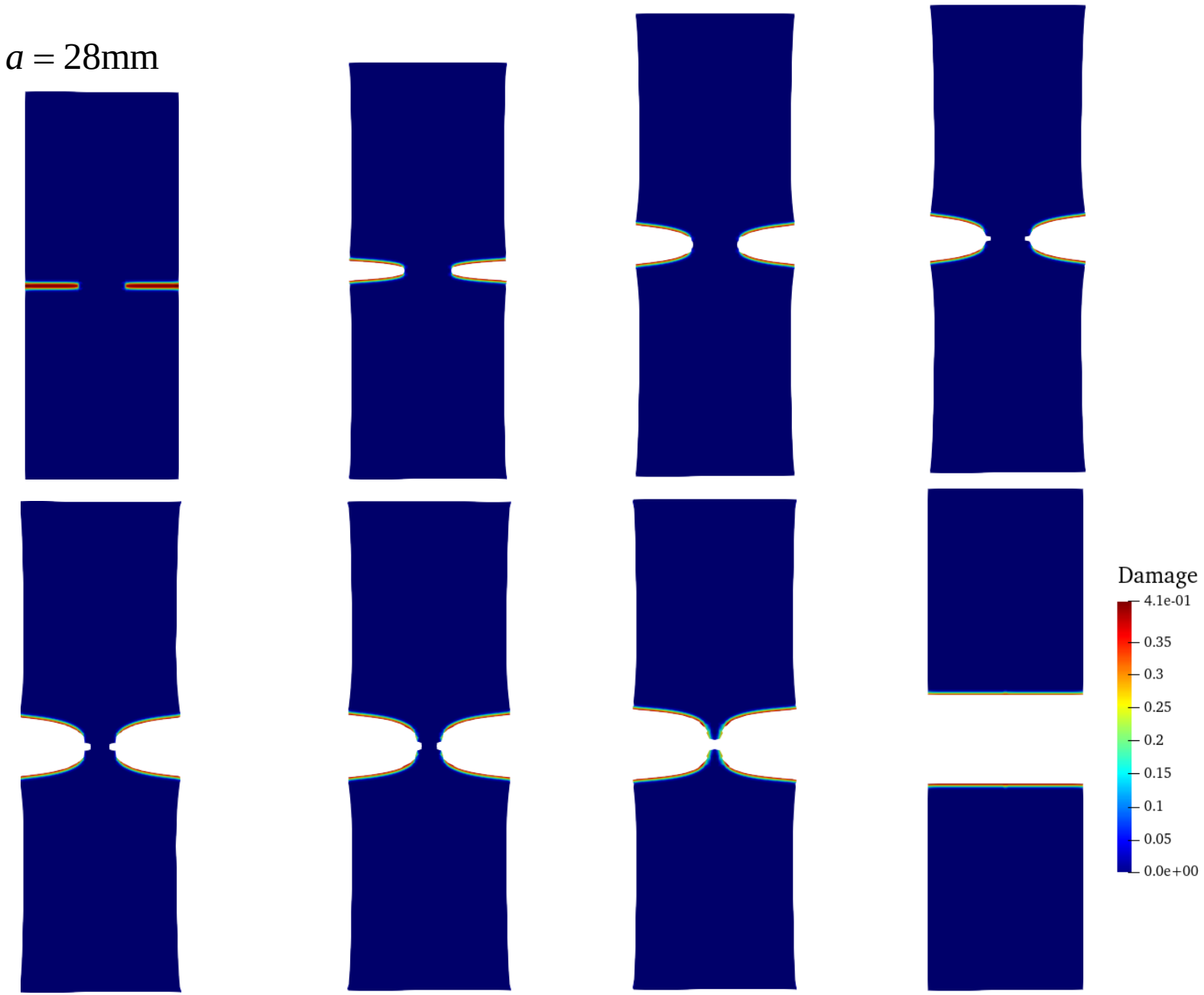

PD
FEM + Phase field
Talamini et al. (2018)
Experiment
Hocine et al. (1998)
$a = 12$mm
Reaction force (N)
Applied displacement (mm)
0
10
20
30
40
0
20
40
60


PD
FEM + Phase field
Talamini et al. (2018)
Experiment
Hocine et al. (1998)
$a = 16$mm
Reaction force (N)
Applied displacement (mm)
0
10
20
30
40
0
20
40
60

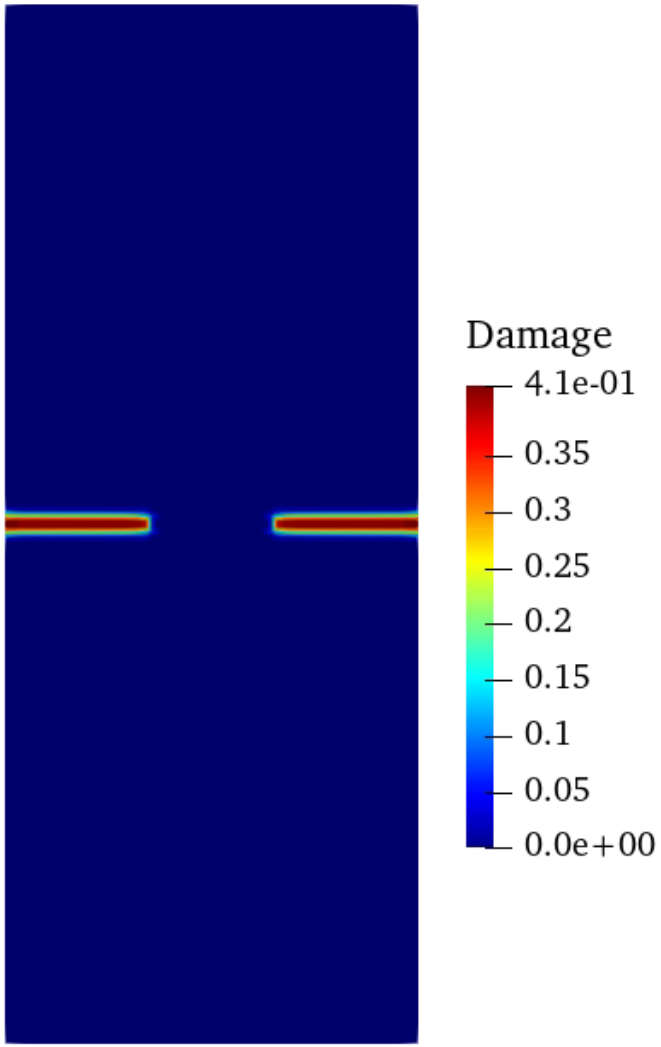

PD
FEM + Phase field
Talamini et al. (2018)
Experiment
Hocine et al. (1998)
$a = 20$mm
Reaction force (N)
Applied displacement (mm)
0
10
20
30
40
0
20
40
60


PD
FEM + Phase field
Talamini et al. (2018)
Experiment
Hocine et al. (1998)
$a = 24$mm
Reaction force (N)
Applied displacement (mm)
0
10
20
30
0
10
20
30
40
50

# Damage initiation, growth and rupture

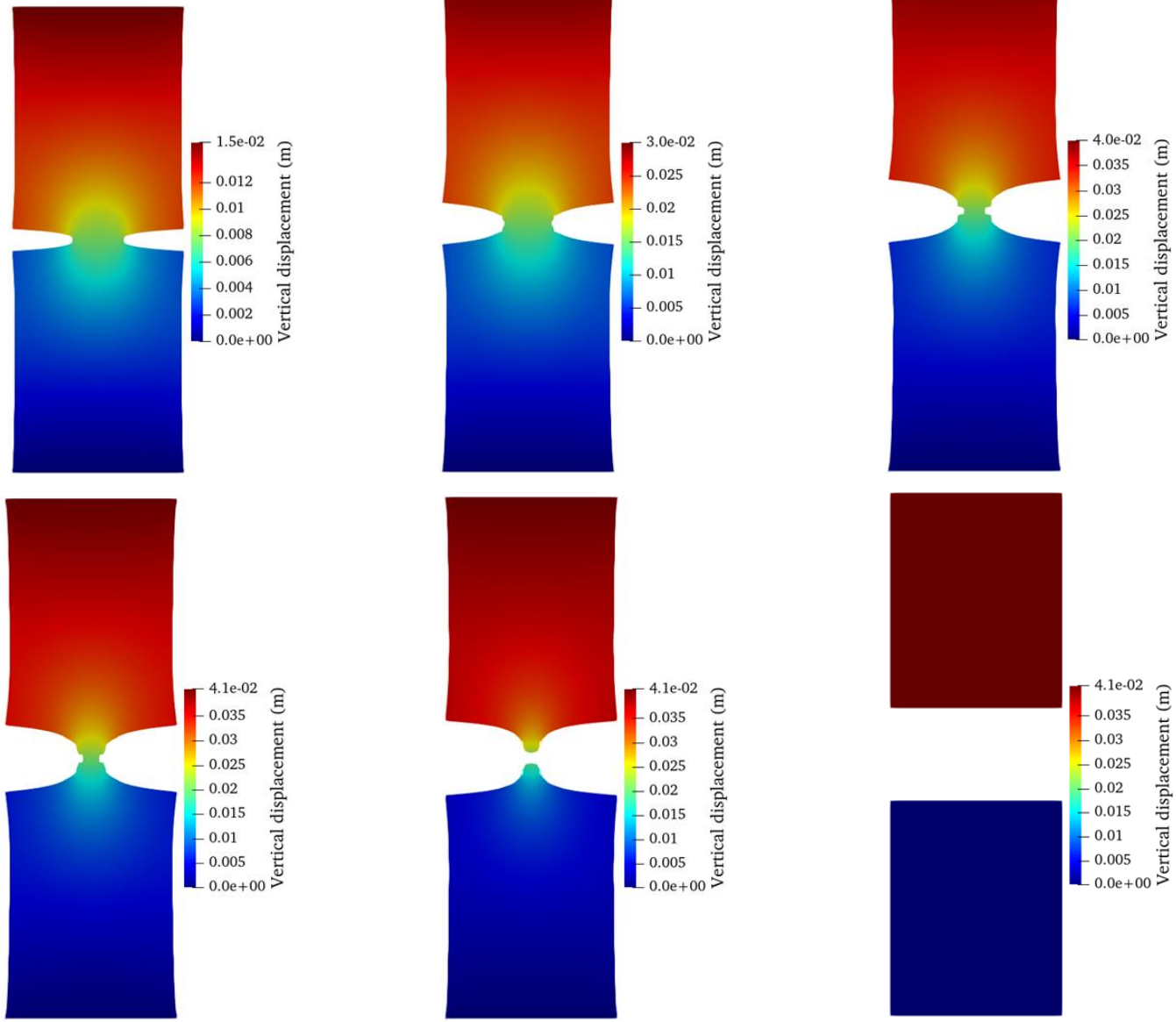


Movie

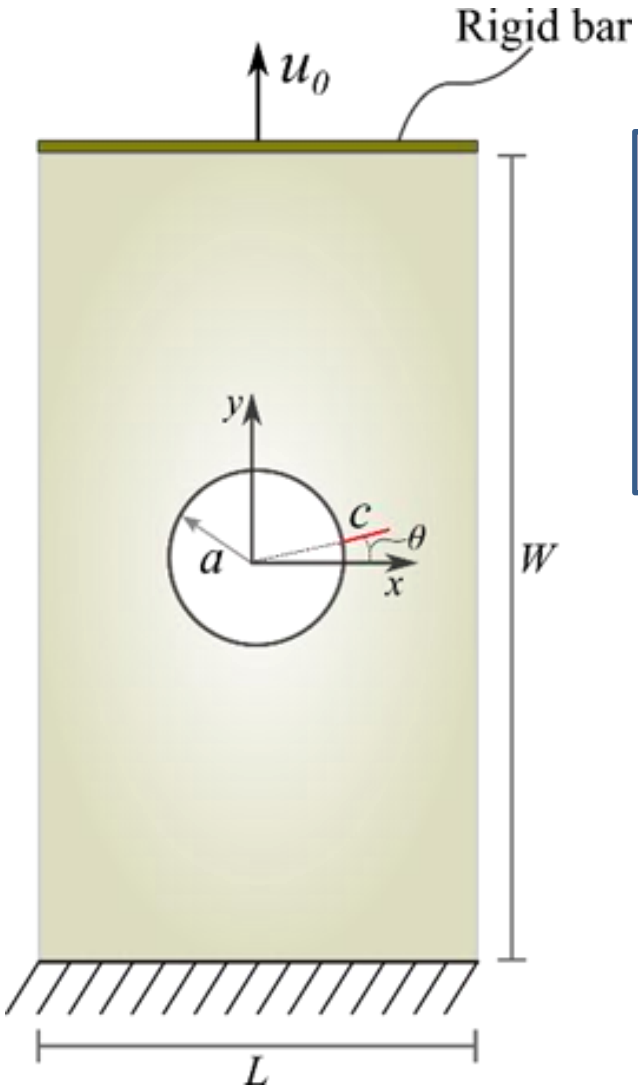


# Vertical displacement as damage propagates

$a = 28\text{mm}$



# Rubber sheet with a radial notch from hole boundary




$L = 80\text{mm}, W = 200\text{mm}, t = 3\text{mm}$

$c = 5\text{ mm},\ \theta = 20°$

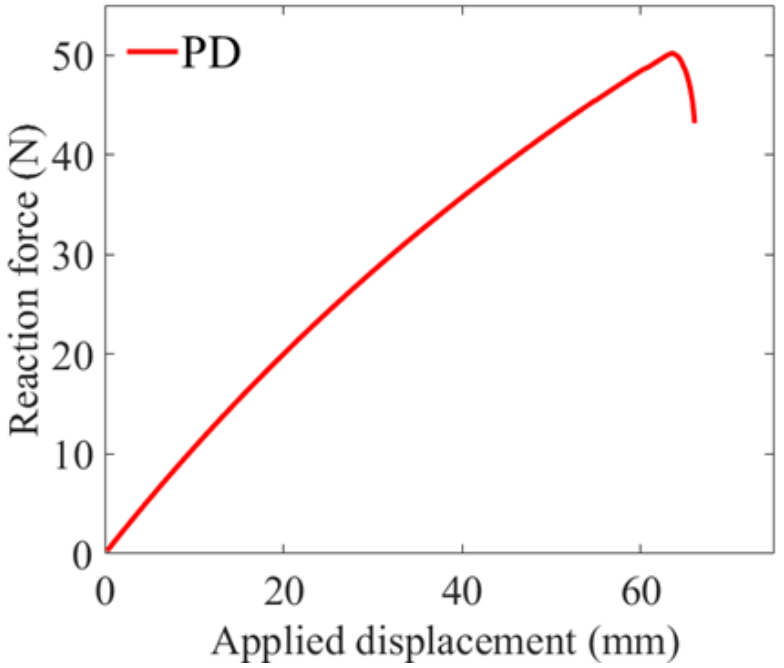
$K = 2.68\text{ MPa},\ \ \mu = 0.268\text{ MPa}$

$\Delta u_0 = 0.25\,\text{mm}$

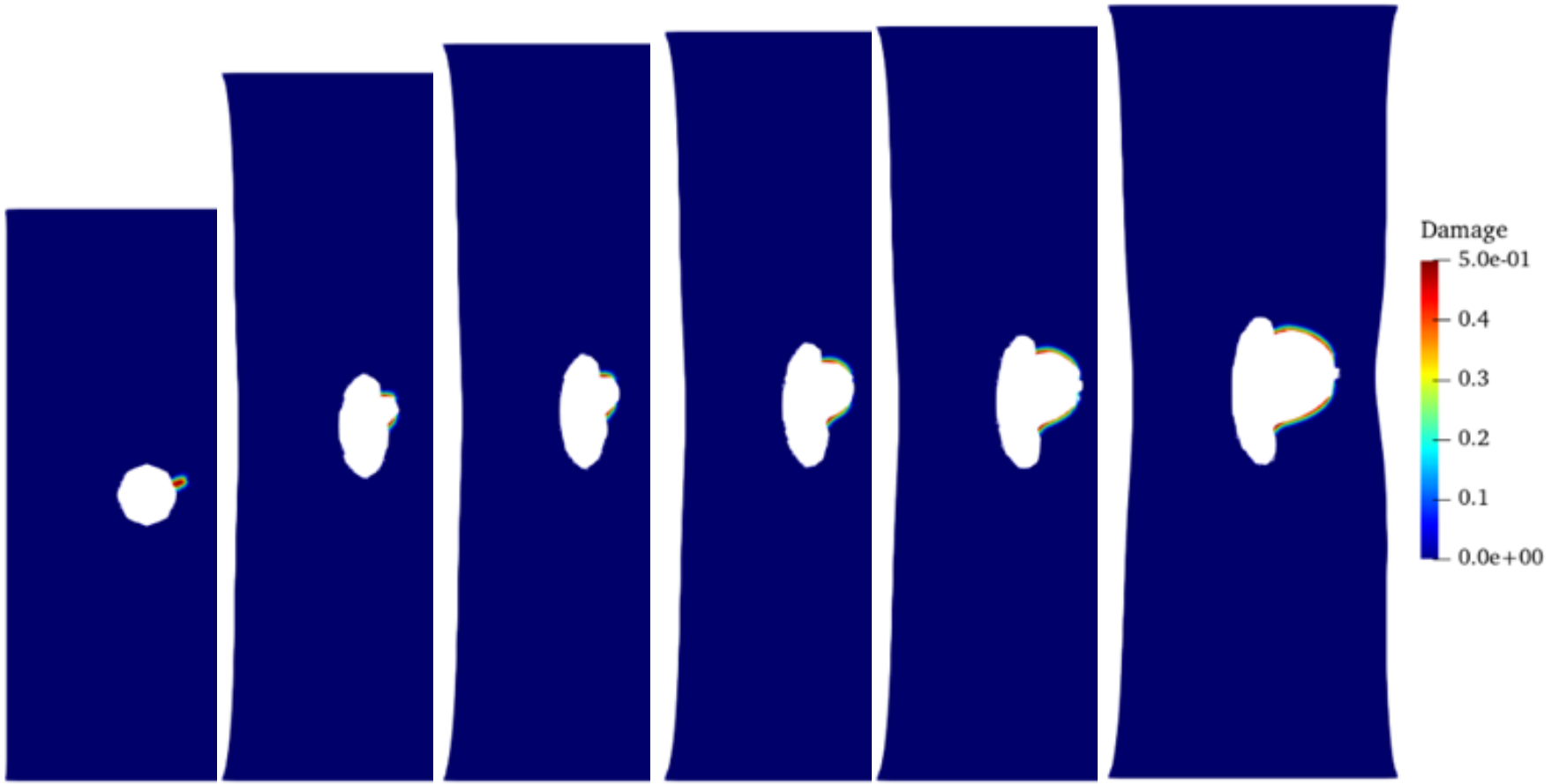
$s_c = 3.1$

$A_{(k)} = 1.0\text{ mm}^2$

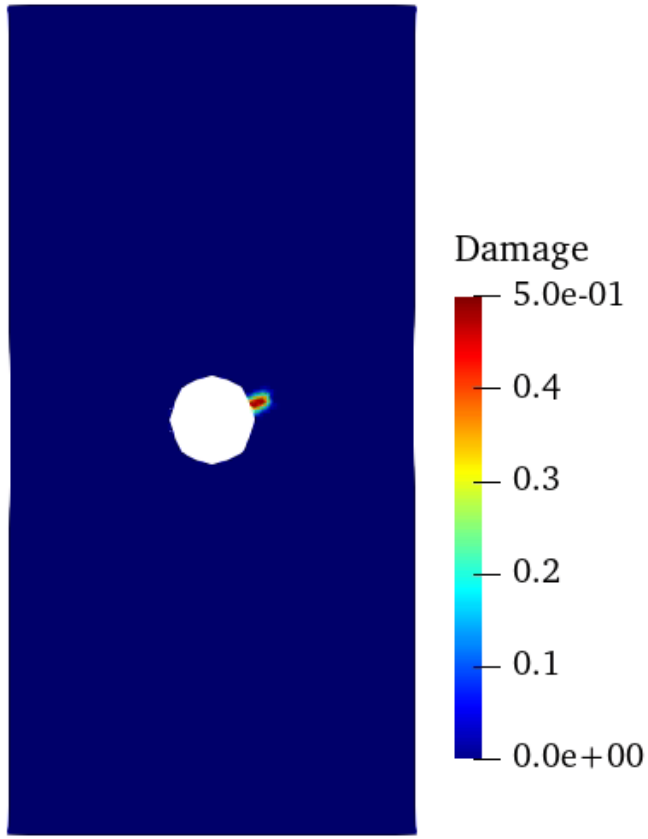
$\delta = 3.015\Delta x$



# Damage initiation and growth



Movie



# Remarks

- Force density vector - bond-associated deformation gradient
  - Free of oscillations and zero energy modes
- PD form of deformation gradient - PDDO
- Weak form of PD governing equations - virtual work
  - Direct imposition of boundary conditions
- Implicit solution
  - Verification against FEA
  - Comparison with experimental results
  - Comparison with FEA coupled with phase field
- PD predictions capture the deformation and failure response