Nonlocal Brittle Fracture Modeling

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Overview of talk

- Global fracture-like defects emerge from Newton's laws applied at each point in the body from nonlocal forces that can soften.
- Spatial localization of gradients emerge from the dynamics.
- In this model the fracture toughness is the same for all choices of nonlocal interaction. This aspect implies the smaller the horizon the smaller the transverse dimension of the softening zone.
- Convergence to a sharp fracture evolution in the small horizon limit.
- Explicit relation between stress power expended on either side of the crack and fracture toughness for double well cohesive peridynamics.
- Nonlocal model recovers the **kinetic relation** for sharp crack growth seen in the modern theory of fracture mechanics given in Anderson, Freund, Ravi-Chandar.

- Peridynamics: Nonlocal modeling of fracture problems: A single field model

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  - Littlewood, D. J. (2010). Simulation of dynamic fracture using peridynamics, finite element modeling, and contact. In *Proceedings of the ASME 2010 International Mechanical Engineering Congress and Exposition (IMECE)*.

Double well force potentials, perydynamics, and numerical methods for nonlocal fracture modeling

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- Jha & Lipton, Plane elastodynamic solutions for running cracks as the limit of double well nonlocal dynamics. arXiv.org ArXiv:1908.07589.
- P. Jha and R. Lipton. Numerical convergence of finite difference approximations for state based peridynamic fracture models. Computer Methods In Applied & Engineering Sciences. 2019, https://doi/org/10.1016/j.cma.2019.03.024
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- P. Jha and R. Lipton. Convergence of state based peridynamic models to LEFM models. To appear in Communications in Applied Mathematics and Computation, 2019.
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- R. Lipton. Dynamic brittle fracture as a small horizon limit of Peridynamics. Journal of Elasticity. 125:1 (2014) 21-50.

Phase field modeling of fracture: A two field model

- Quasi-static:
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- Dynamic:
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  Borden, M., Verhoosel, C., Scott, M., Hughes, T., Landis, C.: A phase-field description of dynamic brittle

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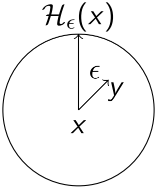
Length scale of nonlocal interaction



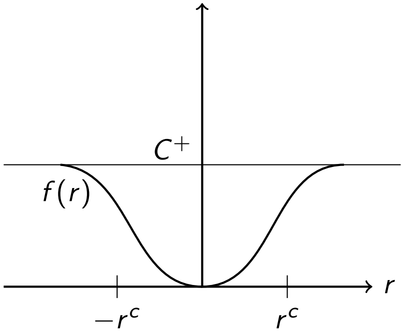
- $\epsilon =$ length scale of nonlocal interaction
- Interaction between x and y only within a ϵ-neighborhood of x
- $\mathcal{H}_\epsilon(x) = \{y \in \mathbb{R}^3 : |y - x| < \epsilon\}$

Displacement and strain

The displacement at x denoted by $u(x)$, and the strain between x and y

$$\mathcal{S} = S(\boldsymbol{y}, \boldsymbol{x}, t; \boldsymbol{u})) = \frac{u(t,y) - u(t,x)}{|y - x|} \cdot e, \text{ where } e = \frac{y-x}{|y-x|} \quad (1)$$

Double Well Potential



Figure

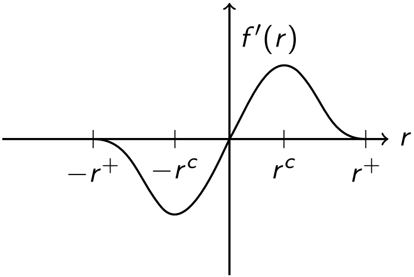
Figure: **The potential function $f(r)$ for tensile force.**

Tensile Potential Parameterized by length scale of nonlocal interaction ϵ

$$\mathcal{W}^{\epsilon}(S(\boldsymbol{y},\boldsymbol{x},t;\boldsymbol{u})) = \frac{J^{\epsilon}(|\boldsymbol{y}-\boldsymbol{x}|)}{\epsilon|\boldsymbol{y}-\boldsymbol{x}|} f(\sqrt{|\boldsymbol{y}-\boldsymbol{x}|}S(\boldsymbol{y},\boldsymbol{x},t;\boldsymbol{u}))$$

$$\mathcal{W}^{\epsilon}(S(\boldsymbol{y},\boldsymbol{x},t;\boldsymbol{u})) \leq \frac{J^{\epsilon}(|\boldsymbol{y}-\boldsymbol{x}|)}{\epsilon} \times |y-x| \times \min\{f'(0)|S(\boldsymbol{y},\boldsymbol{x},t;\boldsymbol{u})|^2, \frac{C^{+}}{|y-x|}\}$$

Force vs tensile strain



(a)

Figure: **Cohesive force.**

Force Parameterized by ϵ

$$\mathcal{L}^{\epsilon}(\boldsymbol{u})(\boldsymbol{x},t) = \frac{2}{\epsilon^d \omega_d} \int_{H_\epsilon(\boldsymbol{x})\cap D} \frac{J^\epsilon(|\boldsymbol{y}-\boldsymbol{x}|)}{\epsilon|\boldsymbol{y}-\boldsymbol{x}|} \partial_S f(\sqrt{|\boldsymbol{y}-\boldsymbol{x}|} S(\boldsymbol{y},\boldsymbol{x},t;\boldsymbol{u})) \boldsymbol{e}_{\boldsymbol{y}-\boldsymbol{x}}\, d\boldsymbol{y}. \tag{2}$$

$$\mathcal{H}_\epsilon(x) = \{y \in \mathbb{R}^3 : |y-x| < \epsilon\},\ x \in D.$$

Dynamics on the domain D

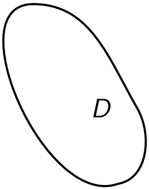


Figure: **Domain D.**

The influence function J satisfies $0 \leq J \leq M$ and $J^\epsilon(|y-x|) = J(|y-x|/\epsilon)$.

The non-local dynamics

$$\rho \ddot{\boldsymbol{u}}^{\epsilon}(\boldsymbol{x}, t) = \mathcal{L}^{\epsilon}(\boldsymbol{u}^{\epsilon})(\boldsymbol{x}, t) + \boldsymbol{b}(\boldsymbol{x}, t), \text{ for } \boldsymbol{x} \in D. \tag{3}$$

Initial conditions on the domain D

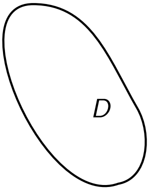


Figure: **Domain D.**

u^ϵ satisfies the initial conditions $u^\epsilon(0,x) = u_0(x)$, $\partial_t u^\epsilon(0,x) = v_0(x)$ for all $x \in D$.

Kalthoff Winkler Experiment Simulation: Trask, Yu, and Parks 2019

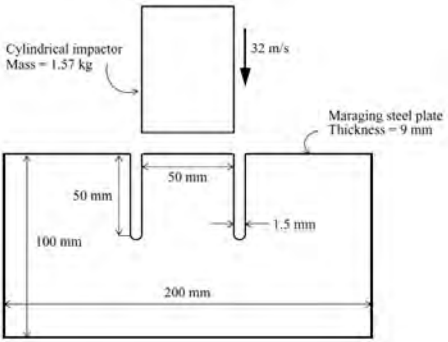


Figure: Experimental set up.

Kalthoff Winkler Experiment Simulation: Trask, Yu, and Parks 2019

Figure: Simulation.

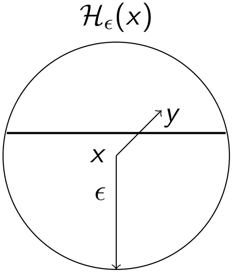
Kalthoff Winkler Experiment Simulation: Trask, Yu, and Parks 2019

Figure: Right quadrant 68° angle crack.

Failure Zone

Definition

Failure Zone. Points **x** where a nonzero fraction of points **y** in horizon centered at **x** have no interaction with **x**.



Fracture toughness definition for tensile loads

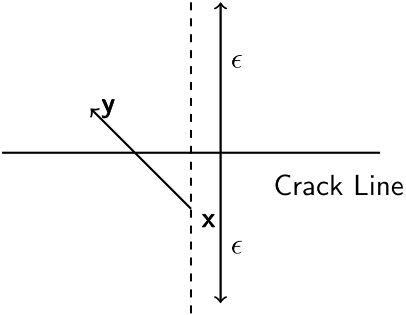


Figure: **Definition of fracture toughness $\mathcal{G}_c$ in 2-d. The work per unit length required to eliminate all interaction between x and y on either side of the line.**

Fracture toughness is the same for all choices of ϵ

- We work with a family of models with the same fracture toughness that are independent of ϵ.
- This means that the region of interaction on both sides of the plane goes to zero with ϵ and we converge to a sharp interface model.
- *Localization as $\epsilon \to 0$ is hard wired into this nonlocal model.*

Failure zone propagating from left to right: A "crack" in the double well cohesive peridynamic model

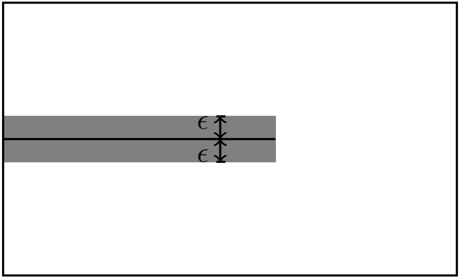


Figure: **Failure zone with centerline**

Models with fracture toughness is the same for all choices of ϵ

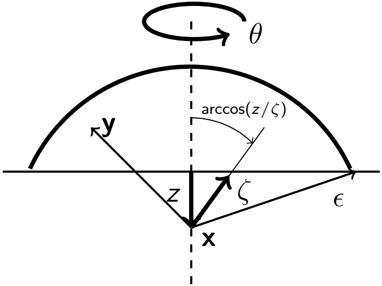


Figure: **Calculation of fracture toughness $\mathcal{G}_c$. For each point x along the dashed line, $0 \leq z \leq \epsilon$, the work required to break the interaction between x and y in the spherical cap is summed up using spherical coordinates centered at x.**

Fracture toughness is the same for all choices of ϵ

$$\mathcal{G}_c(x) = \frac{2}{V_\epsilon} \int_0^\epsilon \int_0^{2\pi} \int_z^\epsilon \int_0^{\arccos\left(\frac{z}{|\zeta|}\right)} J^\epsilon\left(|\zeta|\right) \frac{f_\infty}{\epsilon} |\zeta|^2 \sin\phi \, d\phi \, d|\zeta| \, d\theta \, dz \quad (4)$$

$$= \frac{6}{4} f_\infty \int_0^1 J(r) r^3 dr, \quad (5)$$

Elastic coefficients: tensile and hydrostatic components

Assume a small linear displacement $u(x) = Fx$ over $\mathcal{H}_\epsilon(x)$ & taylor expansion in F in

$$PD^\epsilon(u(t)) = \mathcal{W}^\epsilon(S(\boldsymbol{y}, \boldsymbol{x}, t; \boldsymbol{u})) + \mathcal{V}^\epsilon(\theta(\boldsymbol{x}, t; \boldsymbol{u})) \tag{6}$$

gives to leading order for $\mathcal{S} = Fe \cdot e$ and $|S| << |S_c^\pm|$, $|\theta| << |\theta_c^\pm|$

$$PD^\epsilon(u(t)) = \sum_{ijkl} \left(2\overline{\mu} \frac{\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}}{2} + \overline{\lambda}\delta_{ij}\delta_{kl} \right) F_{ij} F_{kl} \tag{7}$$

Elastic Constants

Here $\overline{\mu}$, $\overline{\lambda}$ are given by the explicit formulas

$$\overline{\mu} = \frac{f''(0)}{10} \int_0^1 |\xi|^3 \omega(|\xi|)\, d|\xi|, \tag{8}$$

and the Lame constant is given by

$$\overline{\lambda} = \frac{f''(0)}{10} \int_0^1 |\xi|^3 \omega(|\xi|)\, d|\xi|. \tag{9}$$

Approaching a local fracture model in the limit of vanishing non-locality

Consider a sequence of solutions u^{ϵ} associated with a sequence of horizons $\epsilon \to 0$.