Theoretical basis

Governing equations for the fluid flow

Moreover, the cubic law is used to evaluate the permeability of the fracture domain:

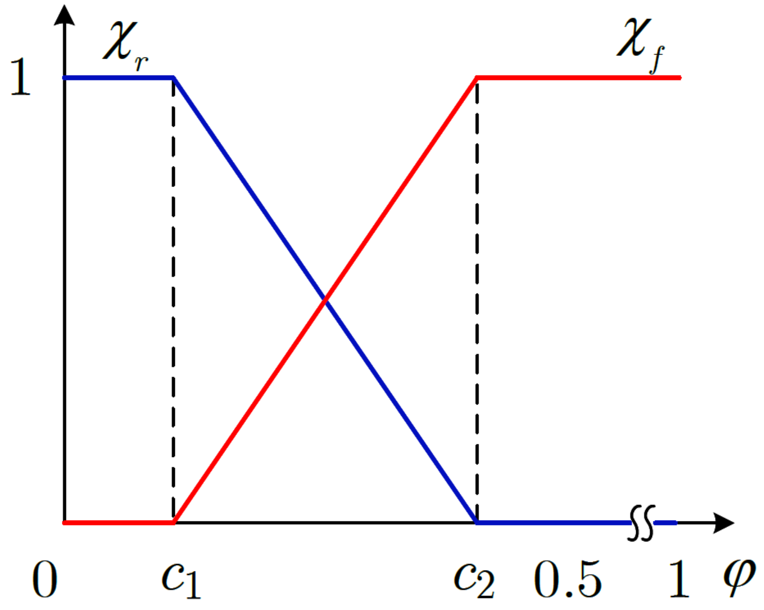
$$k_f = \frac{1}{12}a^2$$

where a is the aperture of the crack.

In the transition domain, two linear functions are used to interpolate the properties of reservoir and fracture domain:

$$\psi_t = \psi_r \chi_r + \psi_f \chi_f$$

where ψ is a generic property.



Discretization and numerical implementation

Peridynamics for the solid skeleton

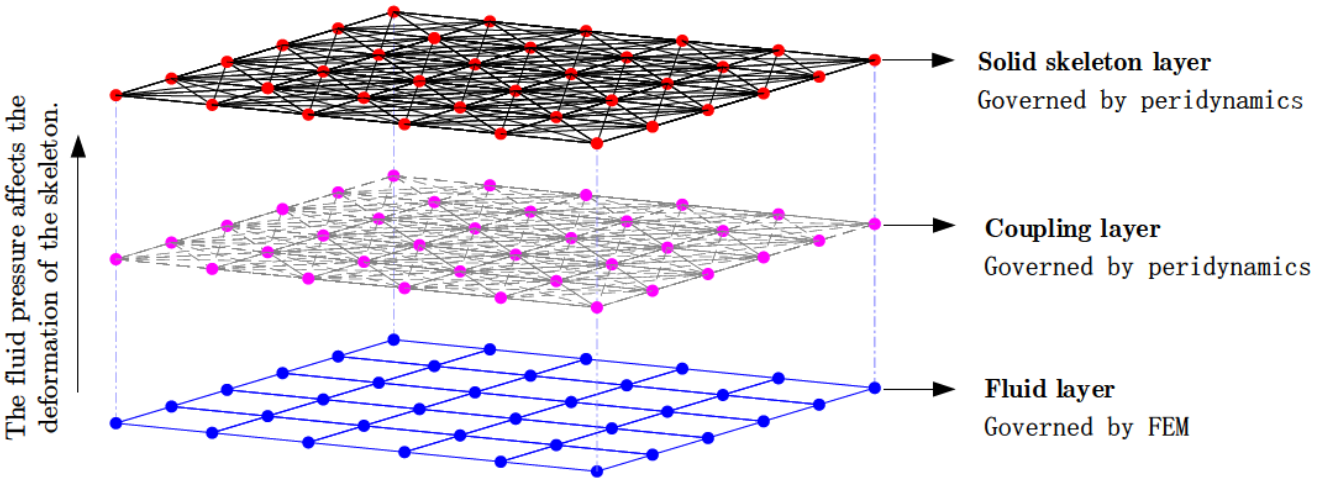
The discretized Peridynamic equations can be written as:

$$\boldsymbol{M}^{PD}\ddot{\boldsymbol{u}} + \boldsymbol{K}^{PD}\boldsymbol{u} - \boldsymbol{Q}^{PD}\boldsymbol{p} = \boldsymbol{f}^{PD}$$

where $\boldsymbol{M}^{PD}$ is lumped mass matrix and the method of obtaining $\boldsymbol{K}^{PD}$ can be found in [1].

$$\boldsymbol{Q}_{ij}^{PD} = 3\alpha \underline{w}\left\langle \boldsymbol{\xi}_{ij} \right\rangle \underline{x}\left\langle \boldsymbol{\xi}_{ij} \right\rangle V_i V_j \begin{bmatrix} \frac{M_x}{m(\boldsymbol{x}_i)} & \frac{M_y}{m(\boldsymbol{x}_i)} & \frac{M_z}{m(\boldsymbol{x}_i)} & -\frac{M_x}{m(\boldsymbol{x}_i)} & -\frac{M_y}{m(\boldsymbol{x}_i)} & -\frac{M_z}{m(\boldsymbol{x}_i)} \\ \frac{M_x}{m(\boldsymbol{x}_j)} & \frac{M_y}{m(\boldsymbol{x}_j)} & \frac{M_z}{m(\boldsymbol{x}_j)} & -\frac{M_x}{m(\boldsymbol{x}_j)} & -\frac{M_y}{m(\boldsymbol{x}_j)} & -\frac{M_z}{m(\boldsymbol{x}_j)} \end{bmatrix}^T$$

[1] Sarego G , Le Q V , Bobaru F , et al. Linearized state-based peridynamics for 2-D problems[J]. International Journal for Numerical Methods in Engineering, 2016:1174-1197.



Discretization and numerical implementation

Finite Element Method for the fluid flow

Using the Galerkin finite element method, the discretized governing equations of the fluid flow assume the following form:

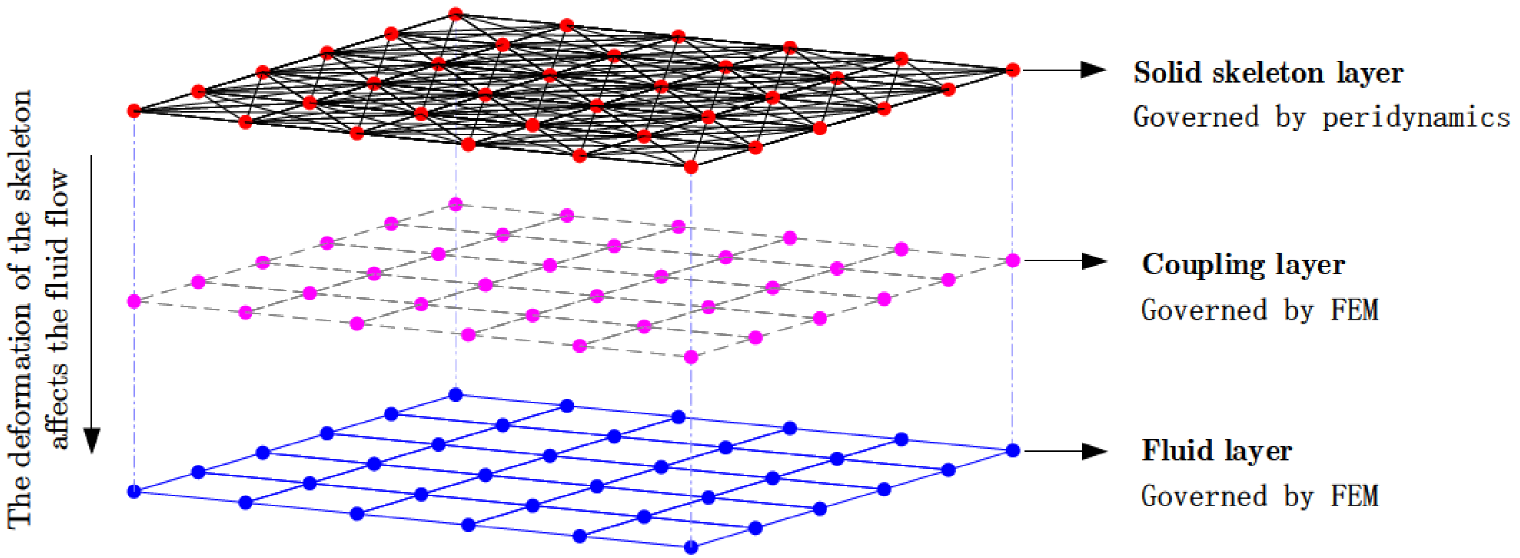
$$\boldsymbol{S}\dot{\boldsymbol{p}} + \boldsymbol{Q}^T\dot{\boldsymbol{u}} + \boldsymbol{H}\boldsymbol{p} = \boldsymbol{q}^w$$

where the matrices can be obtained as:

$$\boldsymbol{S} = \int_\Omega \boldsymbol{N}_p^T s \boldsymbol{N}_p \mathrm{d}\Omega$$

$$\boldsymbol{Q} = \int_\Omega (\boldsymbol{L}\boldsymbol{N}_u)^T \alpha \boldsymbol{m} \boldsymbol{N}_p \mathrm{d}\Omega$$

$$\boldsymbol{H} = \int_\Omega (\nabla \boldsymbol{N}_p)^T \frac{k}{\mu^w} (\nabla \boldsymbol{N}_p) \mathrm{d}\Omega$$



Discretization and numerical implementation

Time discretization

The hydro-mechanical coupled system is given as follows:

$$\begin{bmatrix} \boldsymbol{M}^{PD} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{u}} \\ \ddot{\boldsymbol{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \boldsymbol{Q}^T & \boldsymbol{S} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{u}} \\ \dot{\boldsymbol{p}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}^{PD} & -\boldsymbol{Q}^{PD} \\ \mathbf{0} & \boldsymbol{H} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{p} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{q}^w \end{bmatrix}$$

To avoid solving the combined system with large matrices, the "staggered approach" is adopted:

- Step 1: solve the pressure field ($\boldsymbol{p}^{n+1}$) of the hydraulic diffusion:

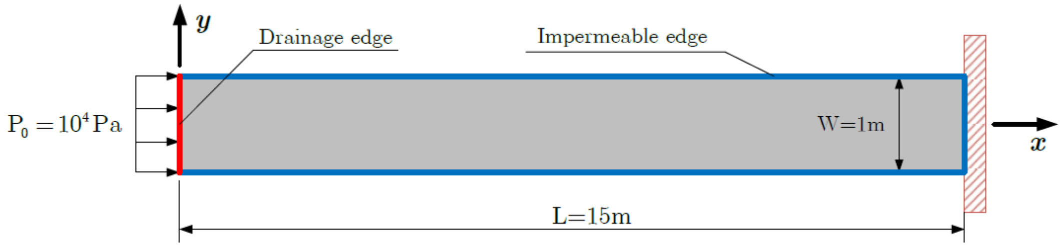
$$\boldsymbol{p}^{n+1} = [\boldsymbol{S} + \vartheta\Delta t\boldsymbol{H}]^{-1}\left\{[\boldsymbol{S} - (1-\vartheta)\Delta t\boldsymbol{H}]\,\boldsymbol{p}^n - \Delta t\boldsymbol{q}^w + \boldsymbol{Q}^T\left(\mathbf{u}^n - \mathbf{u}^{n-1}\right)\right\}$$

- Step 2: solve the displacement field ($\mathbf{u}^{n+1}$) of the peridynamic equation using the adaptive relaxation method

If there are initial or propagating cracks in the porous medium, the permeability and storage matrices (***H*** and ***S***) need to be updated accordingly in each time step.

Numerical verification examples

One-dimensional consolidation problem



Calculation parameters

Young modulus: $E = 10^8\,Pa$;

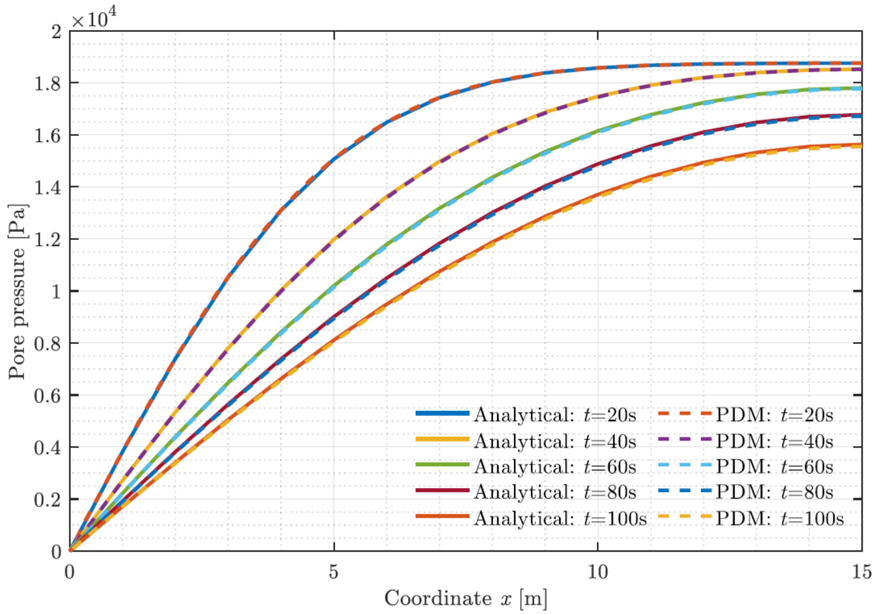
Poisson's ratio: $\nu = 0$;

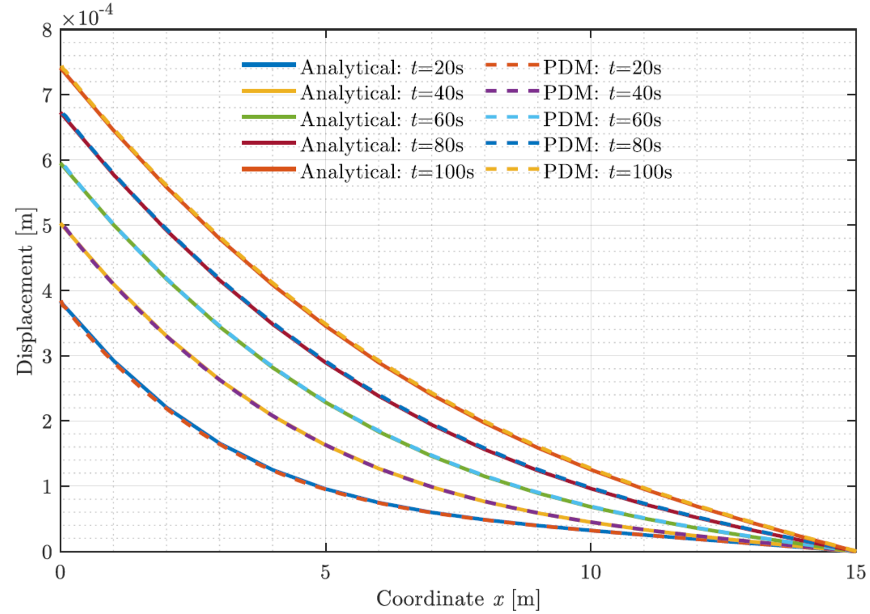
biot constant: $\alpha = 0.5$;

permeability coefficient: $k = 10^{-12}\,m^2$;

fluid viscosity: $\mu_w = 10^{-3}\,Pa \cdot s$;

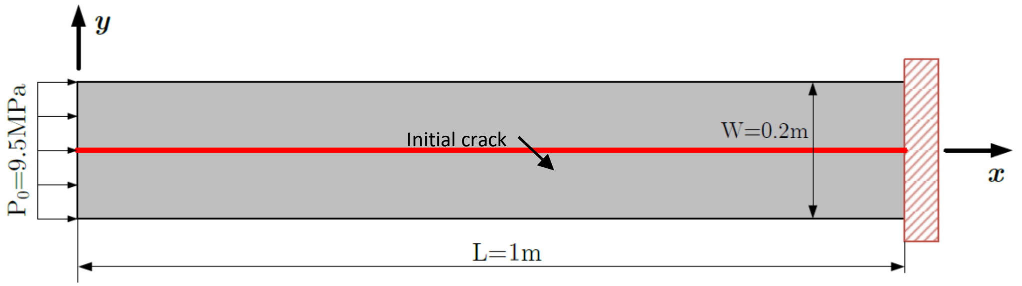
storage coefficient: $S = 1/\,(6.06 \times 10^9\,Pa)$.





Discretization and numerical implementation

Pressure distribution in a single crack



Calculation parameters

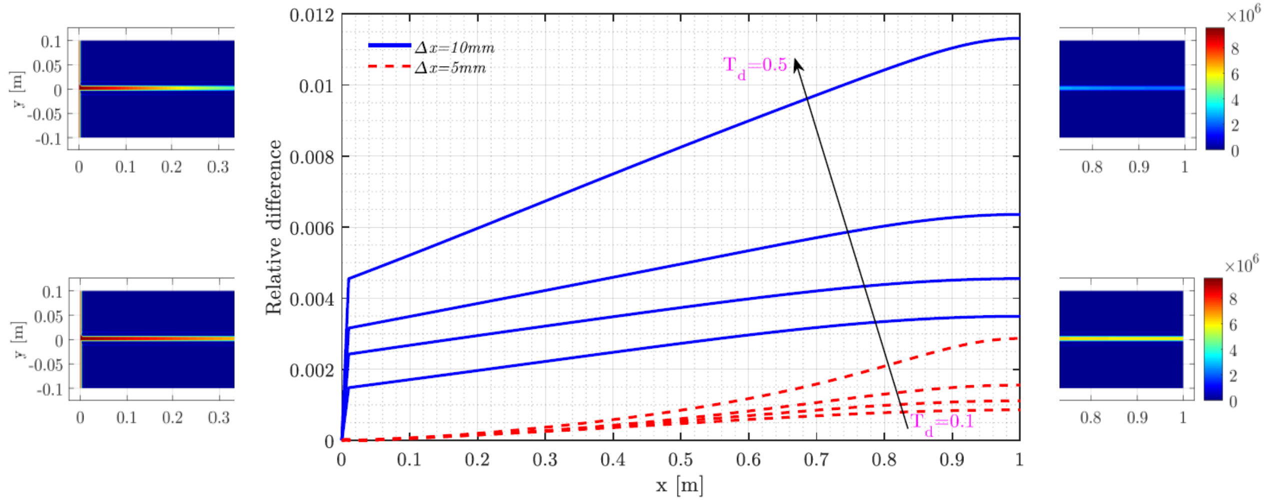
bulk modulus: $K_w = 2.2GPa$;

viscosity coefficient: $\mu_w = 10^{-3} Pa \cdot s$;

biot constant: $\alpha = 1$;

porosity: $n = 2 \times 10^{-5}$;

aperture of the crack: $a = 3 \times 10^{-5} m$.



Hydraulic fracture examples

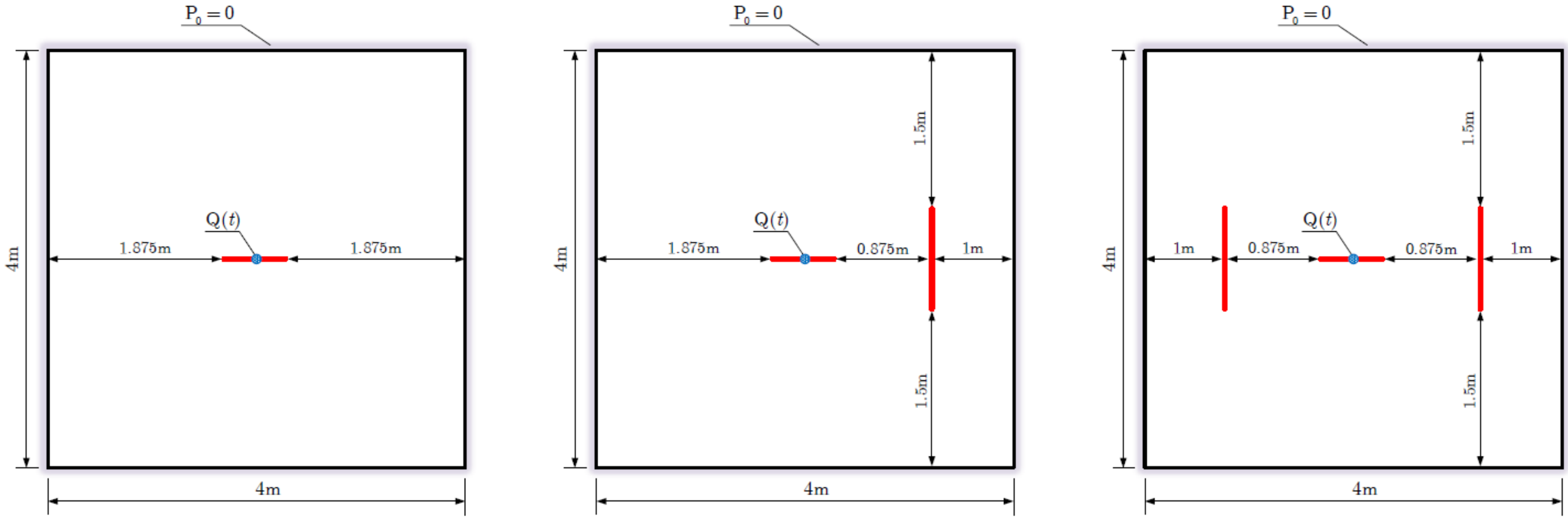
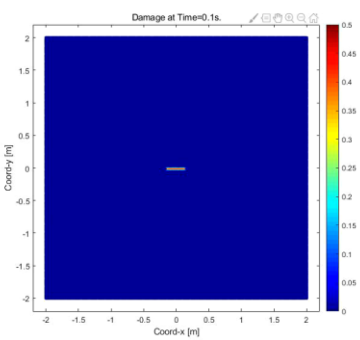


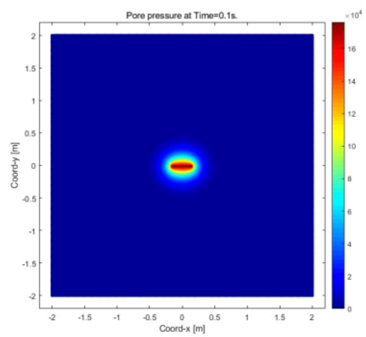
Table 1: Mechanical and fluid parameters used in fluid-driven fracture examples.

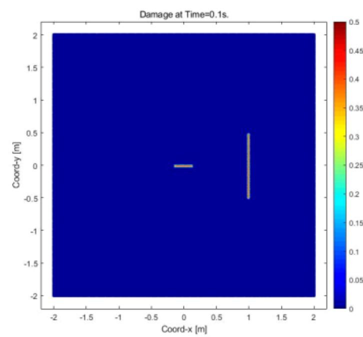
E	υ	G_c	ρ	α	n	K_w	μ	k
$10^8 Pa$	0.2	$100 J/m^2$	$1000 kg/m^3$	1	0.4	$10^8 Pa$	$10^{-3} Pa \cdot s$	$10^{-12} m^2$

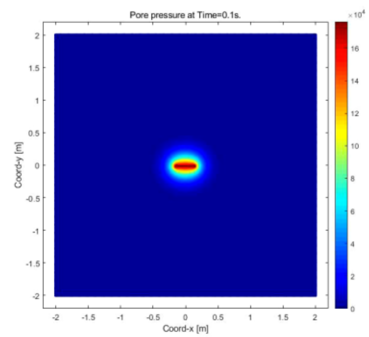
Hydraulic fracture examples

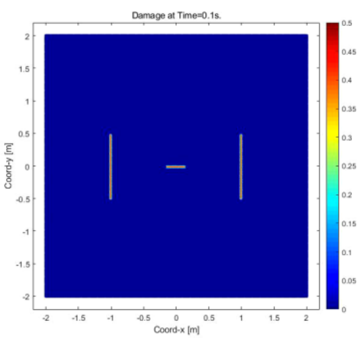
The fluid is injected at the center of the initial crack with a constant volume rate of $Q = 1 \times 10^{-3}\, m^3/s$.

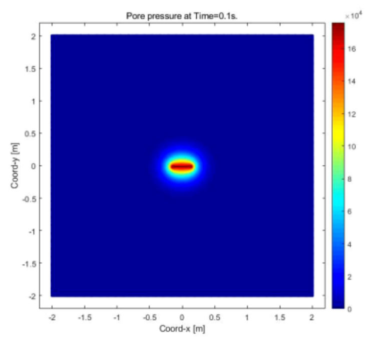


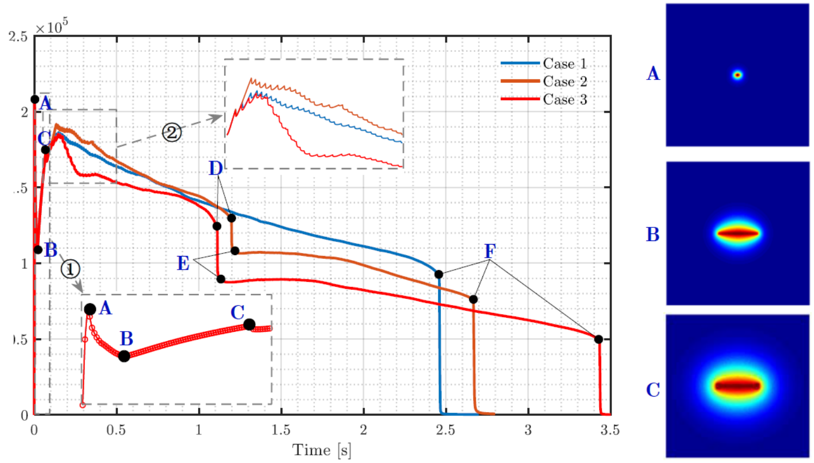






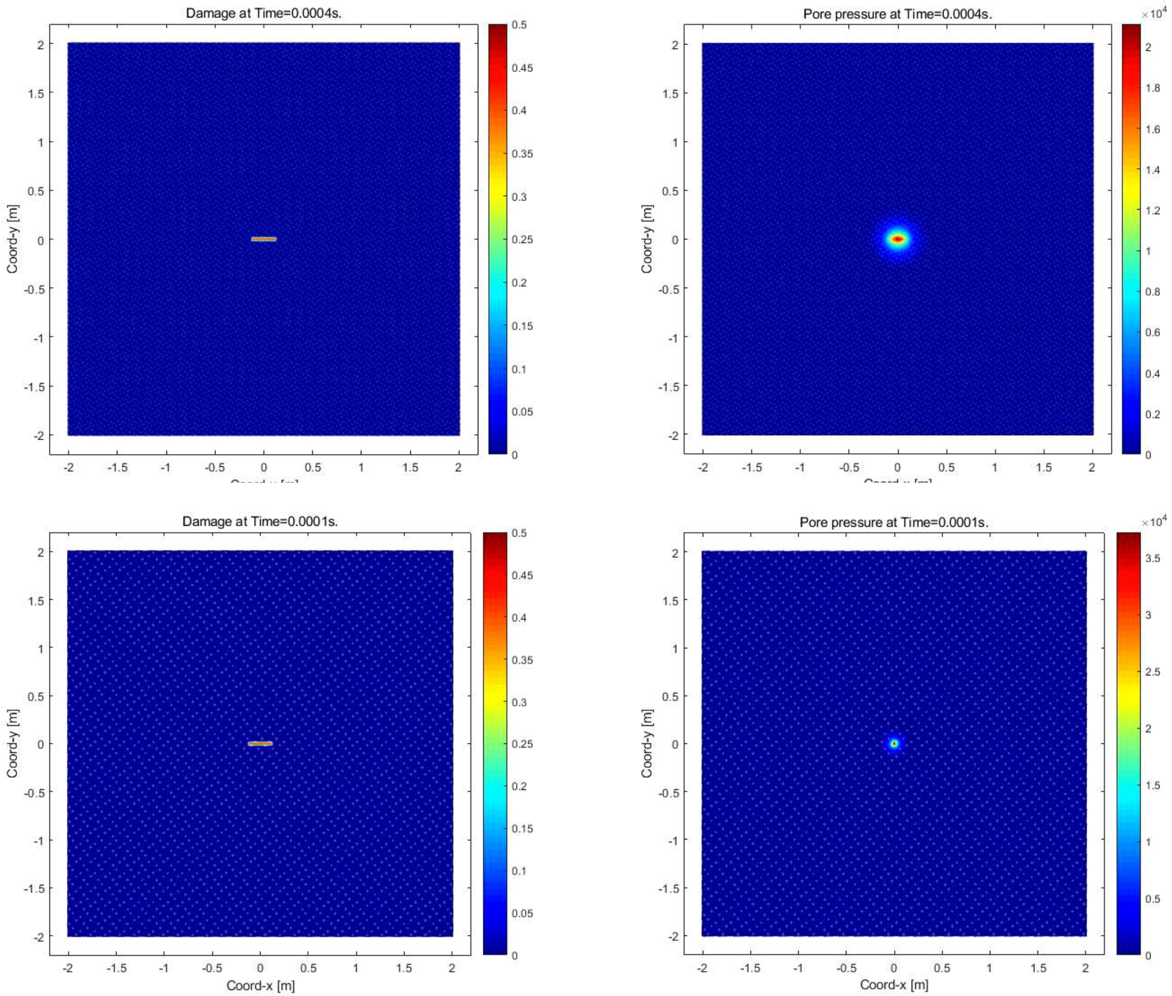






Hydraulic fracture examples

The cases of hydraulic crack propagation and branching



Conclusions

- A coupling between local and non-local theories has been presented to solve multiphysics problems involving crack propagation.
- The proposed model is validated by two benchmark examples of hydraulic consolidation.
- Several examples are presented to demonstrate the capabilities in simulating crack propagation in saturated porous media.
- Similar approaches can be used in the future to effectively simulate crack propagation due to other multiphysics fields (electrical, thermal, chemical, …).
- There is the need to have a better theoretical understanding of what we are doing …

Thank you for your attention

Any questions?

Acknowledgements

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