Workshop on Experimental and Computational Fracture Mechanics
February 26-28, 2020, Baton Rouge, LA

HYBRID FEM AND PERIDYNAMIC SIMULATION OF HYDRAULIC FRACTURE PROPAGATION IN SATURATED POROUS MEDIA

Ugo Galvanetto[1]; Tao Ni[2]; Francesco Pesavento[3]; Mirco Zaccariotto[1]; Francesco Scabbia[1]; Bernhard A Schrefler[3]

[1]Department of Industrial Engineering, University of Padua, Padua, Italy
[2]College of civil and transportation engineering, Hohai University, NanJing, China
[3]Department of Civil Engineering, University of Padua, Padua, Italy

Contents

Multiphysics coupling

- Local and non-local approaches are combined to solve multiphysics problems involving crack propagation:
  - OSB-PD is used for the solid mechanics part
  - FEM is used for other physical fields

- Similar approaches in the past:

- D.Z. Turner 'A coupled local-nonlocal framework for modeling hydraulic fracturing in the Karoo', Research and Applications in Structural Engineering, Mechanics and Computation – Zingoni (Ed.), 2013 Taylor & Francis Group, London.

- R.Wildman, G.A. Gazonas, 'A Multiphysics Finite Element and Peridynamics Model of Dielectric Breakdown', Report number: ARL-TR-8128, U.S. Army Research Laboratory.

Theoretical basis

"States" in OSB-PD

$\underline{x}\langle\xi\rangle$: initial length scalar state

$\underline{y}\langle\xi\rangle$: current length scalar state

$\underline{\boldsymbol{X}}\langle\xi\rangle$: reference vector state

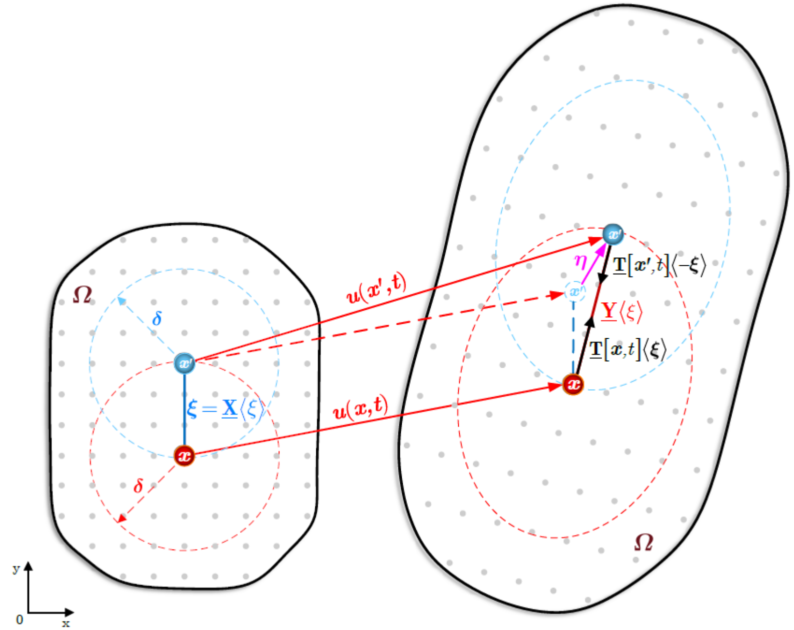
$\underline{\boldsymbol{Y}}\langle\xi\rangle$: deformation vector state

$\underline{\boldsymbol{T}}[x,t]\langle\xi\rangle$: force density vector state of point $\boldsymbol{x}$

$\underline{\boldsymbol{T}}[x',t]\langle-\xi\rangle$: force density vector state of point $\boldsymbol{x}'$

$\underline{\boldsymbol{M}}\langle\xi\rangle$: unit direction vector state

$\underline{e} = \underline{y} - \underline{x}$: extension scalar state



(a) Initial configuration at time t_0 (b) Deformed configuration at time t

Equation of motion of OSB-PD model

$$\rho\ddot{\boldsymbol{u}}(\boldsymbol{x},t) = \int_{H_x} \left\{\underline{\boldsymbol{T}}[\boldsymbol{x},t]\langle\xi\rangle - \underline{\boldsymbol{T}}[\boldsymbol{x}',t]\langle-\xi\rangle\right\} dV' + \boldsymbol{b}(\boldsymbol{x},t)$$

Theoretical basis

OSB-PD poroelastic material

The 3D elastic strain energy of point ***x*** in an OSB-PD material is given as:

$$W\left(\theta, \underline{e}^{d}\right)=\frac{k\theta^{2}}{2}+\frac{15\mu}{2m}\int_{\mathcal{H}_x}\underline{w}\ \underline{e}^{d}\ \underline{e}^{d}\mathrm{d}V_{x'}$$

where k and μ are bulk modulus and shear modulus.

The volume dilatation θ, the deviatoric extension state $\underline{e}^{d}$ and the weighted volume m are defined as:

$$\theta=\frac{3}{m}\int_{\mathcal{H}_x}\left(\underline{w}\ \underline{x}\ \underline{e}\right)\mathrm{d}V_{x'} \qquad \underline{e}^{d}=\underline{e}-\frac{\theta\underline{x}}{3} \qquad m=\int_{\mathcal{H}_x}\underline{w}\left\|\boldsymbol{\xi}\right\|^{2}\mathrm{d}V_{x'}$$

in which $\underline{w}$ is an influence function here defined as: $\underline{w}=\exp\left(-\frac{\|\boldsymbol{\xi}\|^{2}}{\delta^{2}}\right)$

Based on the above notions, the force density scalar state is defined as:

$$\underline{t}=3k\theta\frac{\underline{w}\ \underline{x}}{m}+15\mu\underline{e}^{d}\frac{\underline{w}}{m}$$

Then the force density vector state $\underline{\mathbf{T}}$ in the equation of motion is given as:

$$\underline{\mathbf{T}}=\underline{t}\cdot\underline{\boldsymbol{M}}$$

Theoretical basis

OSB-PD poroelastic material

Dilatational term

Deviatoric term

$$\underline{t} = 3k\theta\frac{\underline{w}\,\underline{x}}{m} + 15\mu\underline{e}^{d}\frac{\underline{w}}{m}$$

Volume dilatation value

$$\theta = \left(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}\right)$$

For small-strain CCM

Volume stress

$$k\theta$$

$$\Downarrow$$

$$\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right)/3$$

Effective stress principle

Adapted to PD by DZ Turner

$$\underline{t}^{tot} = 3\left(k\theta - \alpha p\right)\frac{\underline{w}\,\underline{x}}{m} + 15\mu\underline{e}^{d}\frac{\underline{w}}{m}$$

Theoretical basis

OSB-PD failure criterion (taken from BB-PD)

The "bond stretch" criterion is adopted to assess the bond failure status:

$$\underline{\varrho}\langle\boldsymbol{\xi}\rangle = \begin{cases} 1, & \text{if } s\langle\boldsymbol{\xi}\rangle < s_c \\ 0, & \text{otherwise} \end{cases}$$

where the bond stretch s and the critical stretch s_c are defined as:

$$s\langle\boldsymbol{\xi}\rangle = \frac{\underline{e}\langle\boldsymbol{\xi}\rangle}{\underline{x}\langle\boldsymbol{\xi}\rangle} \qquad s_c = \sqrt{\frac{5G_c}{6E\delta}}$$

where G_c is the critical energy release rate for mode I fracture, E is the elastic modulus and δ is the horizon length.

The damage value at point $\boldsymbol{x}$ in the system is defined as:

$$\varphi = 1 - \frac{\int_{\mathcal{H}_{\boldsymbol{x}}} \underline{w}\ \underline{\varrho}\ \mathrm{d}V_{\boldsymbol{x}'}}{\int_{\mathcal{H}_{\boldsymbol{x}}} \underline{w}\ \mathrm{d}V_{\boldsymbol{x}'}}$$

The cracks can be identified wherever $\varphi \geq 0.5$.

Theoretical basis

OSB-PD crack aperture

For later use, an algorithm to evaluate aperture values at the PD nodes on the crack surface is developed.

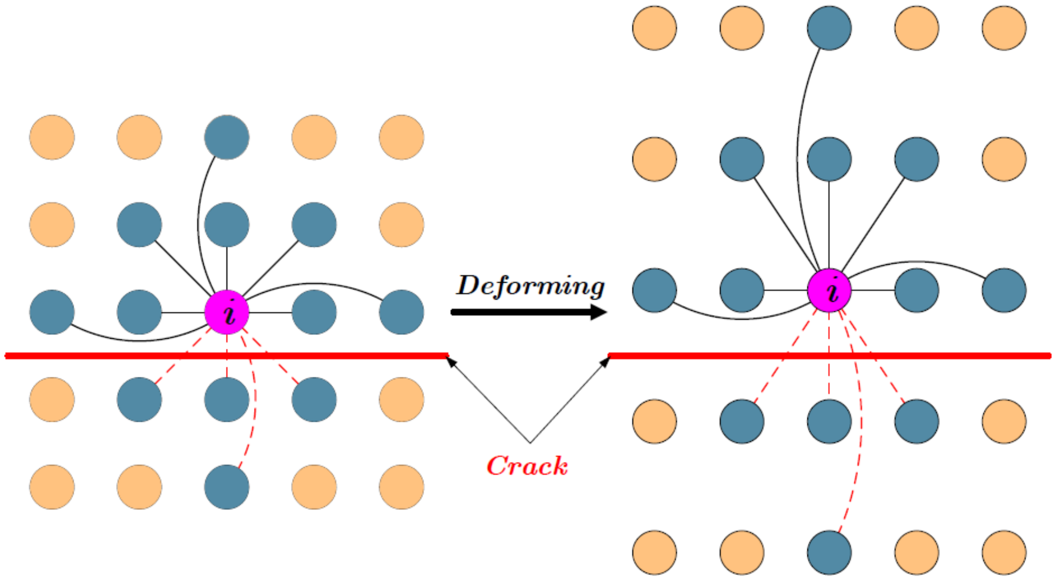
The algorithm considers all broken bonds connected with node *i* and, for each of them, evaluates the aperture in the initial direction of the bond:

$$a_b = \underline{y}\cos\beta - \underline{x}$$

Then the crack aperture of node *i* is the average of bond apertures:

$$a = \left(\sum_{b=1}^{n_b} a_b\right)/n_b$$

where n_b is the number of broken bonds connected with node *i*.



Theoretical basis

Governing equations for the fluid flow

To formulate the flow governing equations, the whole domain is divided into three parts relying on two damage threshold values [(1)]:

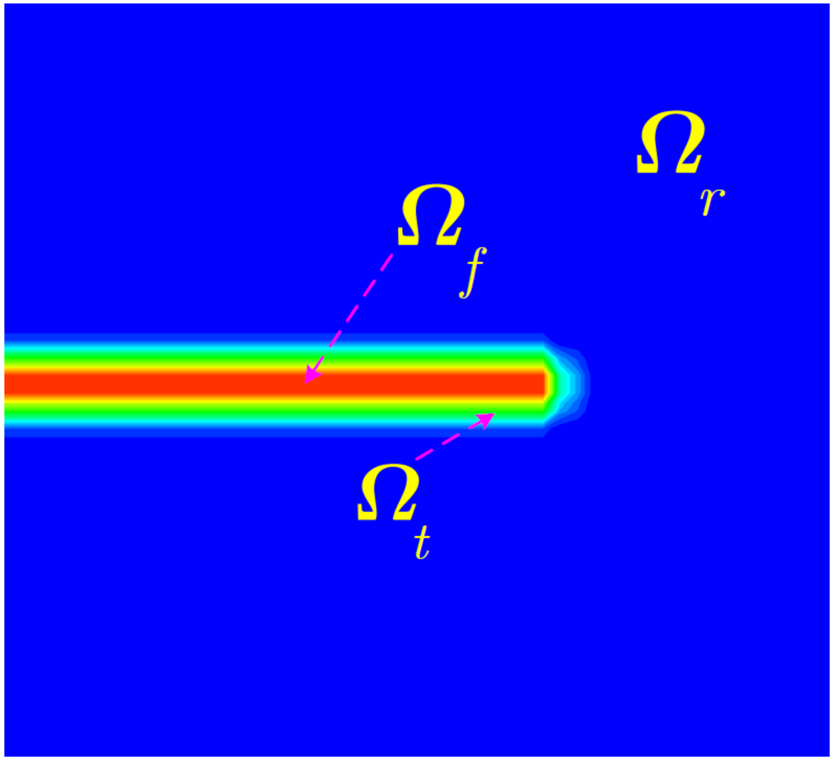
$\varphi \leq c_1 \longrightarrow$ Reservoir domain Ω_r

$\varphi \geq c_2 \longrightarrow$ Fracture domain Ω_f

$c_1 \leq \varphi \leq c_2 \longrightarrow$ Transition domain Ω_t

Threshold values:

$c_1 = 0.2 \qquad c_2 = 0.35$ (m=3)



[(1)]Inspired by Shuwei Zhou, Xiaoying Zhuang, Timon Rabczuk, 'Phase-field modeling of fluid-driven dynamic cracking in porous media', Comput. Methods Appl. Mech. Engrg. 350 (2019) 169–198.

Theoretical basis

Governing equations for the fluid flow

Darcy's law is used to describe the flow field in the saturated porous media and classical Biot poroelasticity theory is adopted.
Then the governing equations of the fluid flow are the following:

In reservoir domain
$$s_r \frac{\partial p}{\partial t} + \alpha_r \frac{\partial \varepsilon_{vol}}{\partial t} + \nabla \cdot \left[\frac{k_r}{\mu} \left(-\nabla p + \rho_r g \right) \right] = \frac{q_r}{\rho_r}$$

In fracture domain
$$s_f \frac{\partial p}{\partial t} + \nabla \cdot \left[\frac{k_f}{\mu} \left(-\nabla p + \rho_f g \right) \right] = \frac{q_f}{\rho_f}$$

In transition domain
$$s_t \frac{\partial p}{\partial t} + \alpha_t \frac{\partial \varepsilon_{vol}}{\partial t} + \nabla \cdot \left[\frac{k_t}{\mu} \left(-\nabla p + \rho_t g \right) \right] = \frac{q_t}{\rho_t}$$