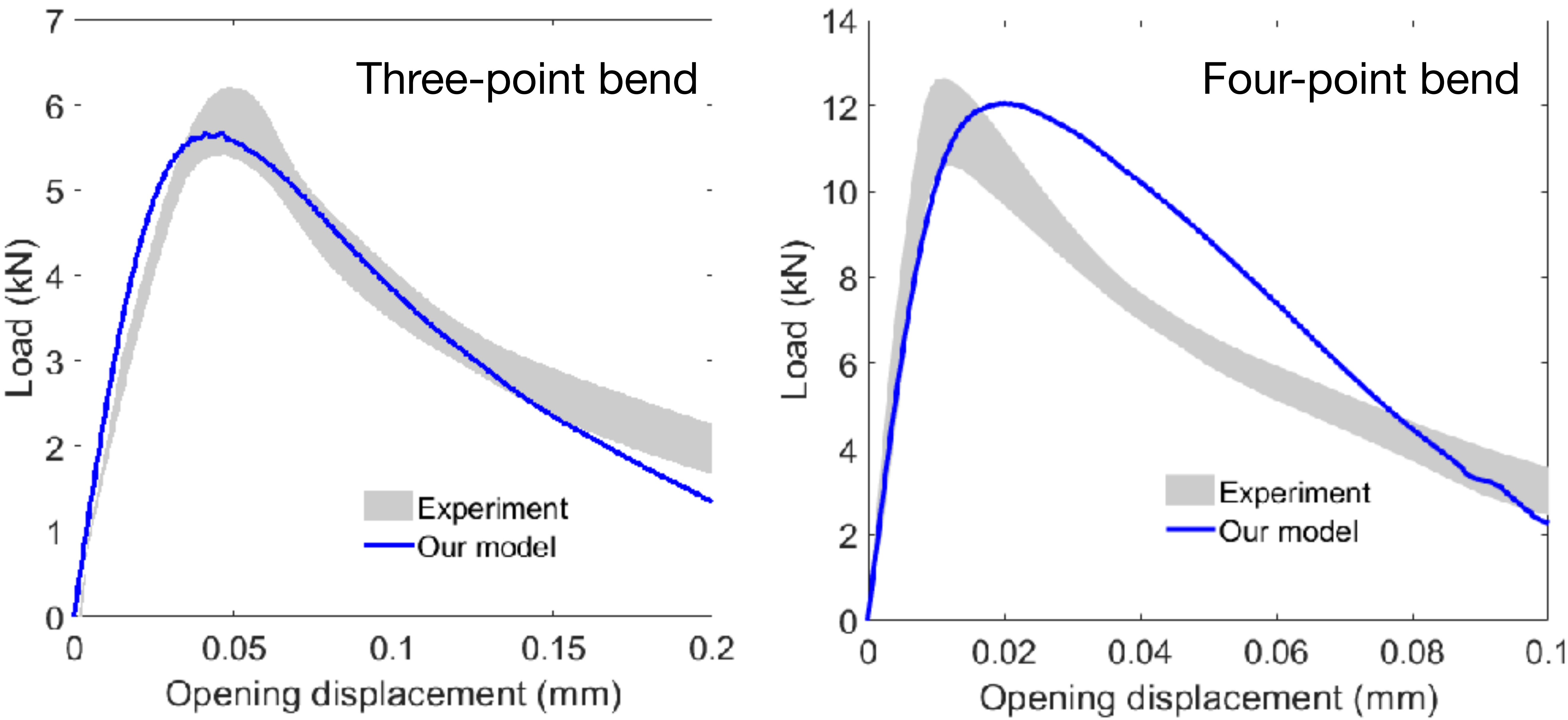
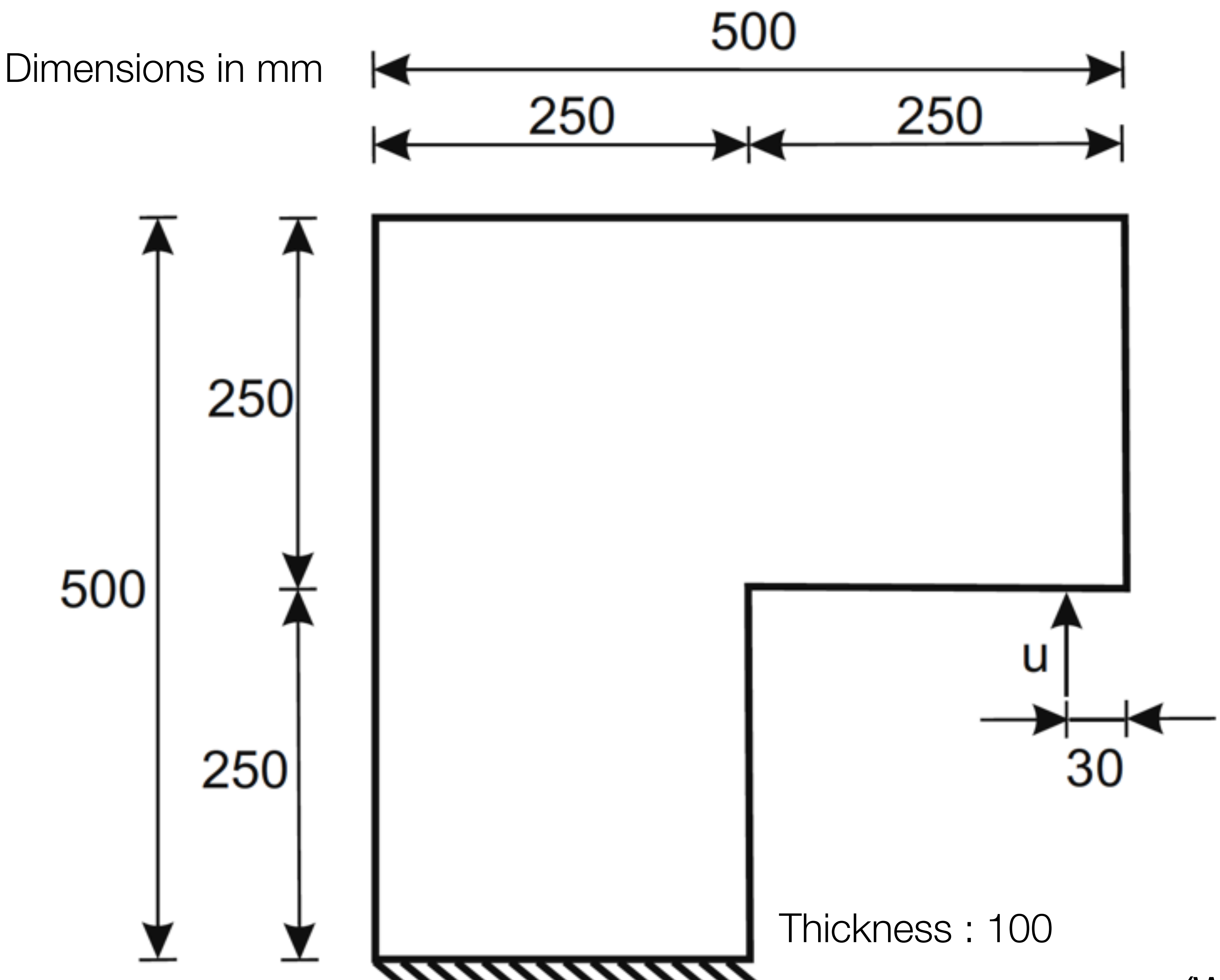
Asymmetric bend tests : Load-displacement comparison



Material Parameters

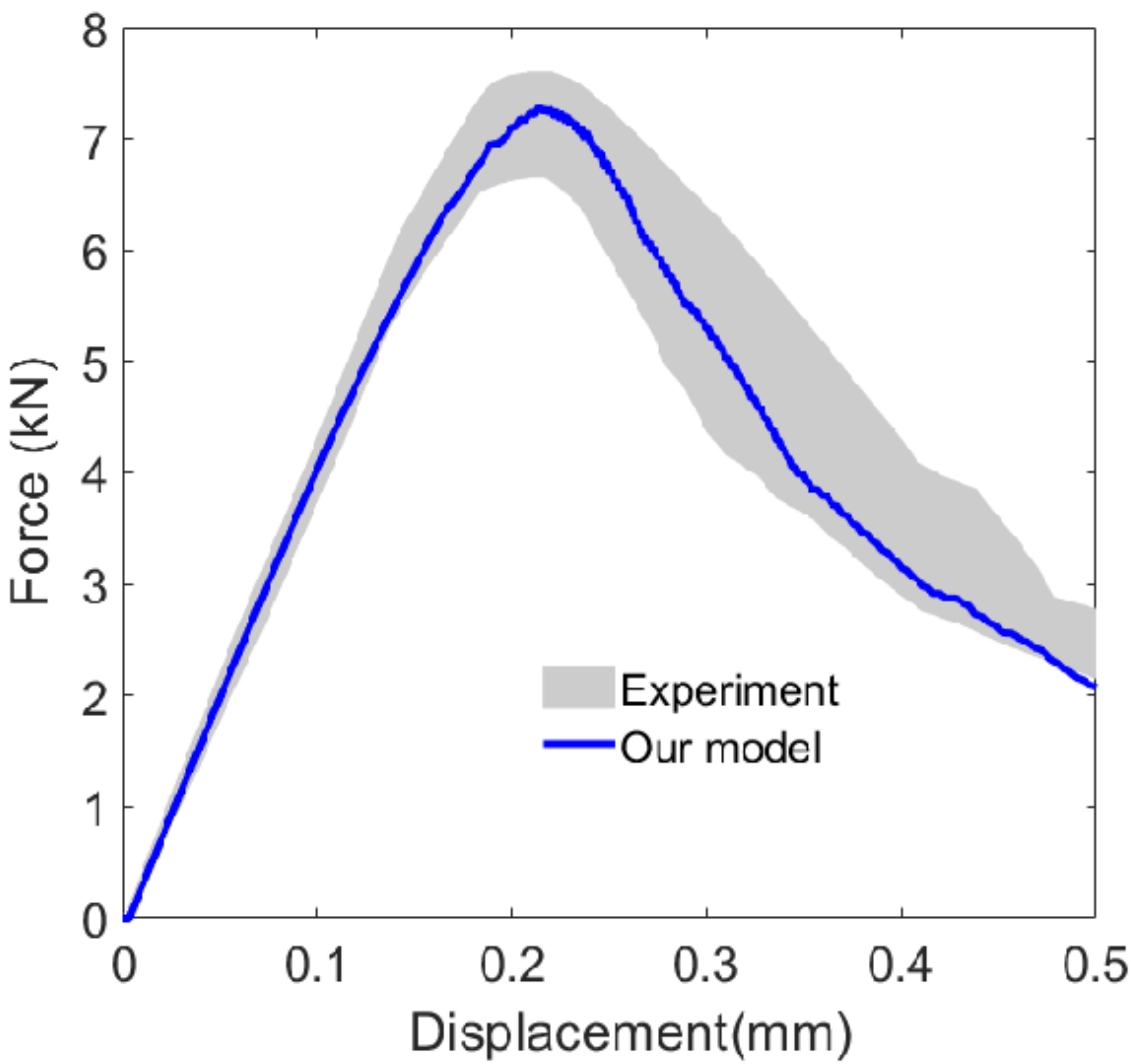
E	ν	S^c	$\varkappa$	ϵ^c_{crit}	ψ_*	ℓ	ζ
38	0.2	3.1	0.7	2.5x 10^{-4}	2.1	3	40
GPa	-	MPa	-	-	kJ/m ^{3}	mm	kPa-s

L-shaped panel test



(Winkler, 2001)

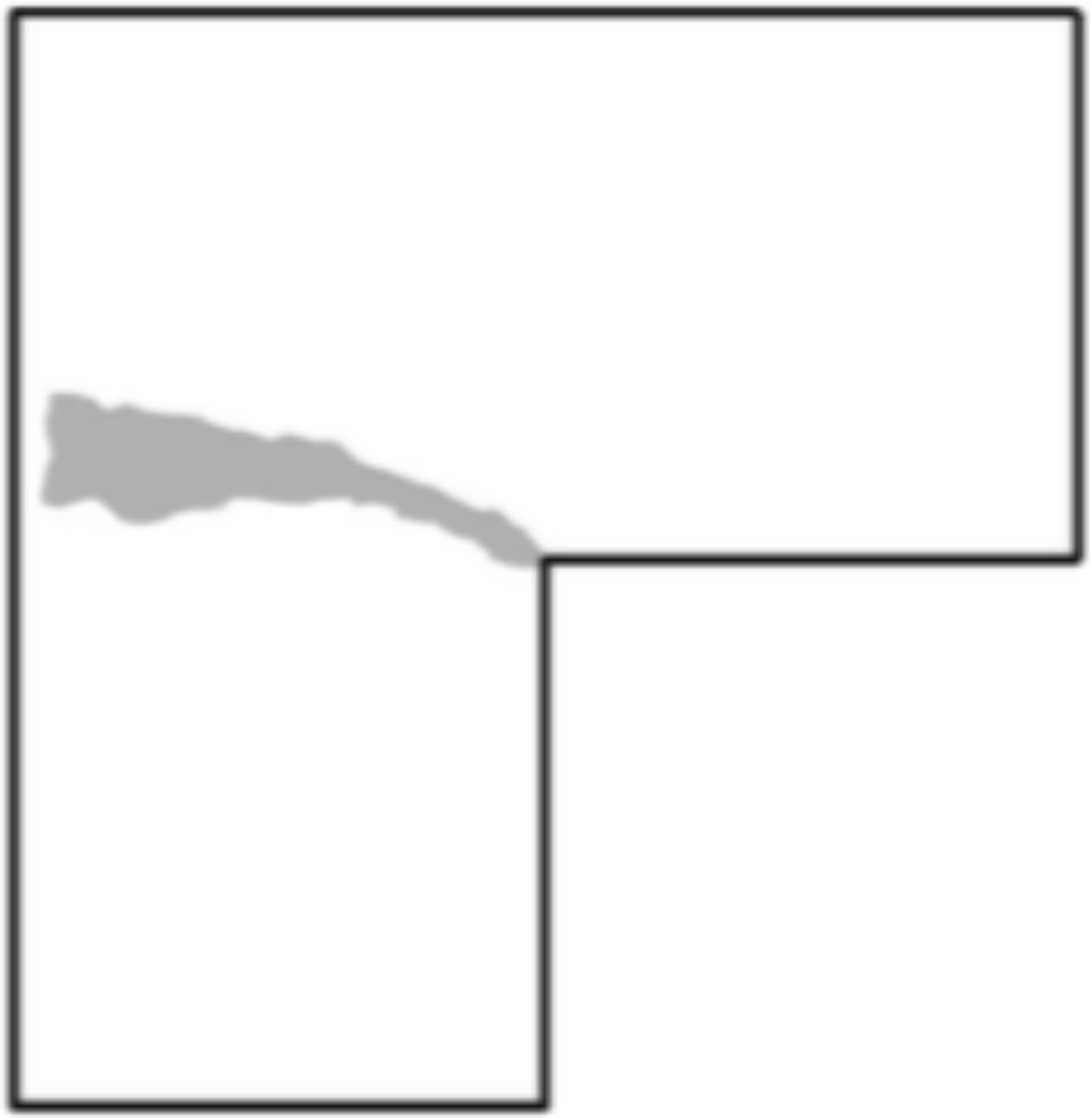
L-shaped panel test



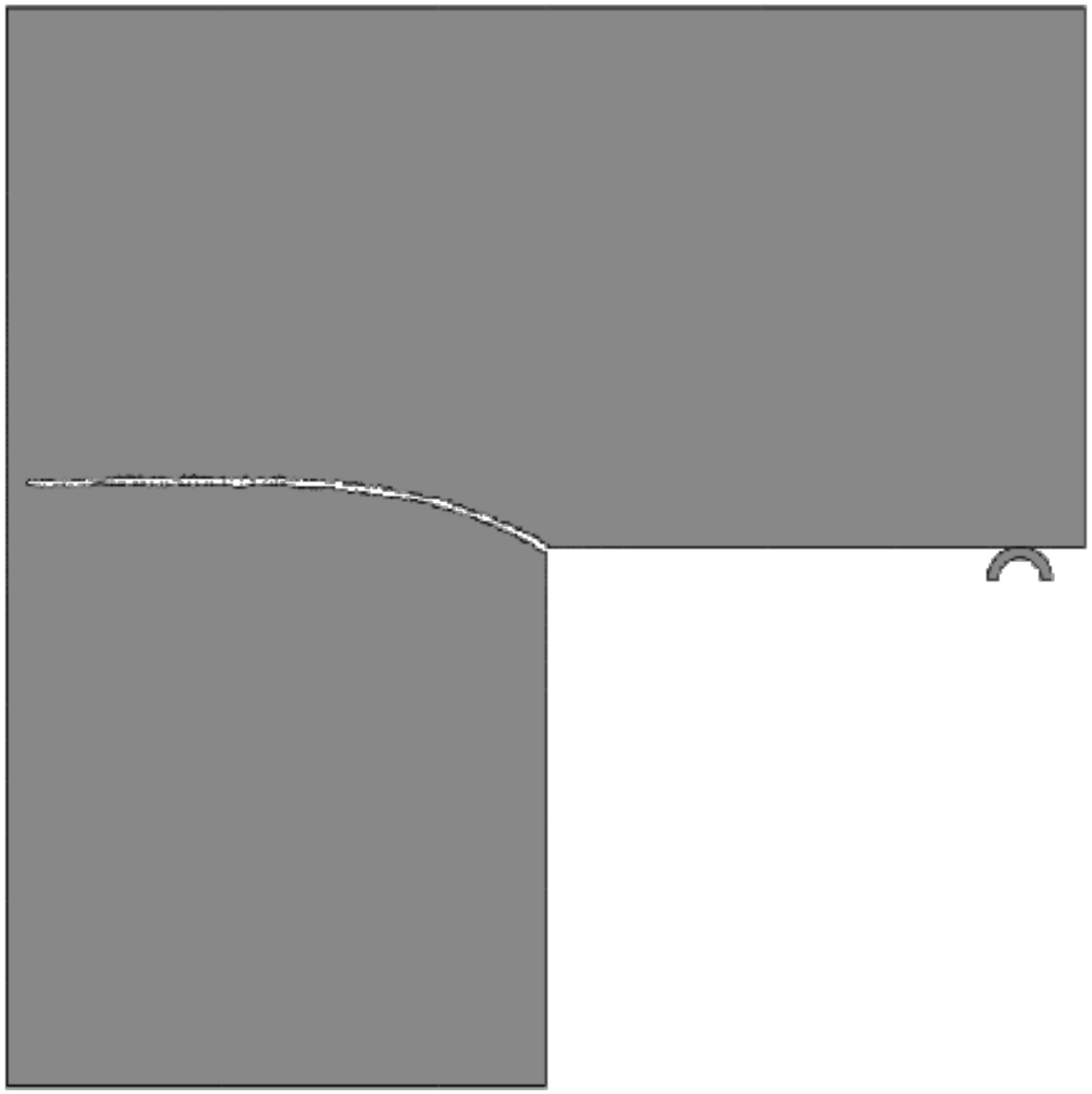
Material Parameters

E	18	GPa
ν	0.18	-
S^c	2.5	MPa
$\varkappa$	0.7	-
ϵ^c_{crit}	4.9×10^{-4}	-
ψ_*	3.2	kJ/m ^{3}
ℓ	4	mm
ζ	40	kPa-s

L-shaped panel test

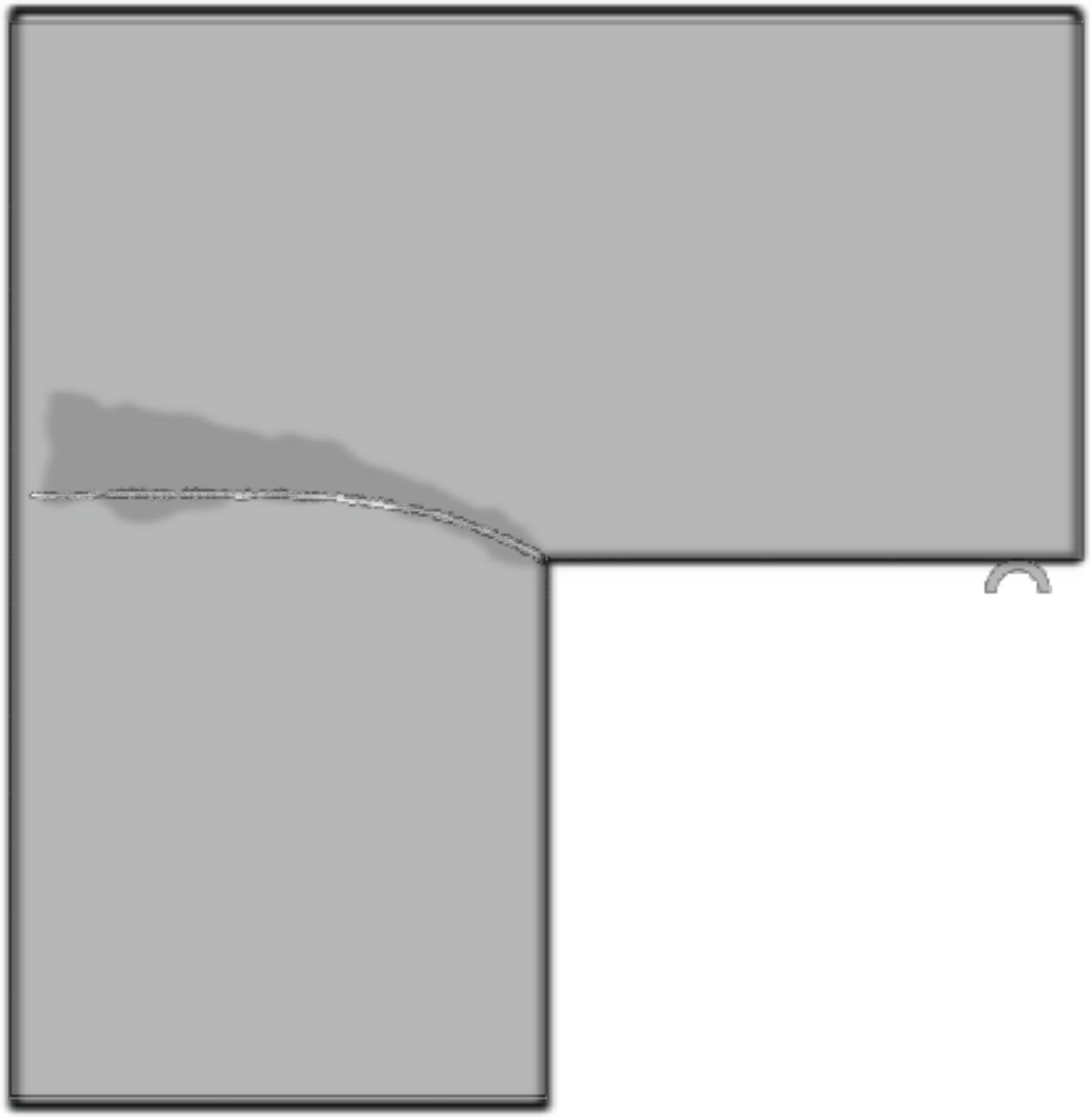


Experimentally observed crack pattern



Phase-field crack (our model)

L-shaped panel test



Phase field crack path overlaid against the experimentally observed crack pattern

Concluding remarks

- We have formulated a new gradient-damage theory for modeling quasi-brittle fracture of concrete.
- Our theory goes beyond the existing phase-field theories for brittle fracture in that the theory:
  - allows for some amount of craze-type inelasticity prior to damage initiation, and that
  - it overcomes the need to decompose the energy into positive and negative contributions.
- The theory has been implemented numerically in ABAQUS as a user element subroutine (UEL).
- The theory has reasonably good predictive capabilities.
- Much more needs to be done.

A gradient-damage theory for fracture of elastomeric materials

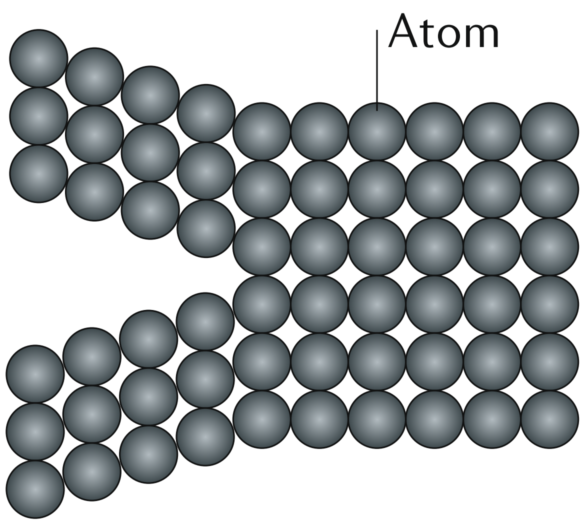
Lallit Anand
with
Yunwei Mao and Brandon Talamini

WFM 2020

Massachusetts Institute of Technology

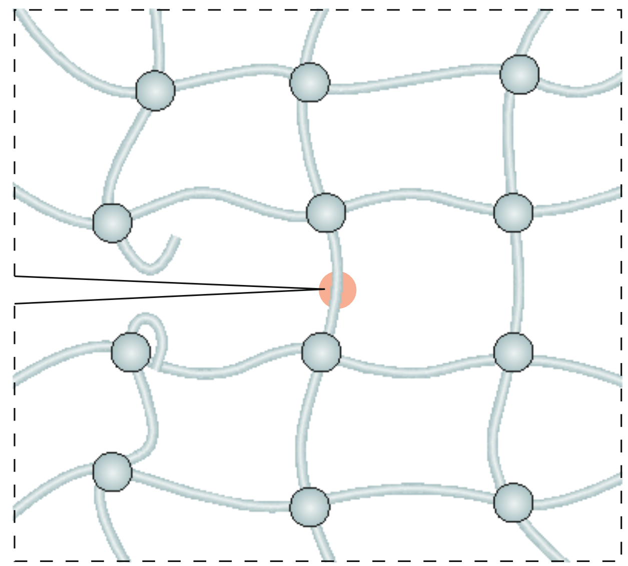
Fracture of elastomers

Griffith, 1921

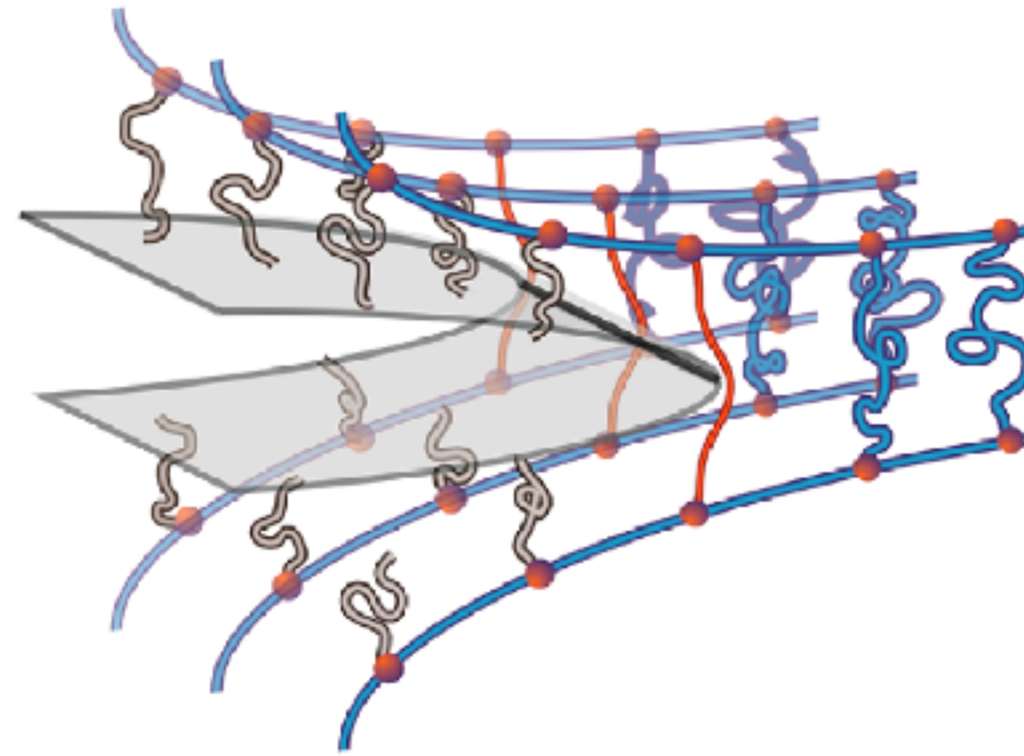


Crystalline materials
Break a layer of atoms
Toughness ~1 J/m^2

Lake-Thomas 1967



Elastomeric materials
Snap a layer of chains
Toughness ~10-1000 J/m^2



(From Yang & Suo, 2018)

- **Ideal** fracture by chain-scission of elastomeric materials with strong covalent crosslinks — in the spirit of Lake and Thomas (1967)
- Neglect any viscoelasticity or Mullin-type effects.

Modeling deformation of elastomeric materials

- For an isotropic material at a temperature ϑ the free-energy function is a symmetric function of the principal stretches λ_i $(i = 1, 2, 3)$:

$$\psi_{\mathrm{R}} = \hat{\psi}(\lambda_1, \lambda_2, \lambda_3, \vartheta), \qquad J = \lambda_1 \lambda_2 \lambda_3 = 1.$$

- **Effective stretch**,

$$\bar{\lambda} \stackrel{\text{def}}{=} \frac{1}{\sqrt{3}} \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2},$$

and consider a special free-energy,

$$\hat{\psi}_{\mathrm{R}}(\bar{\lambda}; \vartheta).$$

- Since $\quad \psi_{\mathrm{R}} = \varepsilon_{\mathrm{R}} - \vartheta\, \eta_{\mathrm{R}},$

$$\psi_{\mathrm{R}}(\bar{\lambda}, \vartheta) = \varepsilon_{\mathrm{R}}(\bar{\lambda}, \vartheta) - \vartheta\, \eta_{\mathrm{R}}(\bar{\lambda}, \vartheta).$$

- For elastomeric materials the **internal energy is classically assumed to be independent of stretch** and a function of the temperature only,

$$\varepsilon_{\mathrm{R}} = \hat{\varepsilon}_{\mathrm{R}}(\vartheta)\,,$$

and that **the entropy is a separable function of temperature and the effective stretch**

$$\eta_{\mathrm{R}} = f(\vartheta) + g(\bar{\lambda}).$$

Arruda-Boyce free energy function

- A widely used free energy function is

$$\psi_{\mathrm{R}} = -\theta\eta_{\mathrm{R}} = G_0\,\lambda_L^2\left[\left(\frac{\bar{\lambda}}{\lambda_L}\right)\beta + \ln\left(\frac{\beta}{\sinh\beta}\right)\right], \qquad \beta \stackrel{\text{def}}{=} \mathcal{L}^{-1}\left(\frac{\bar{\lambda}}{\lambda_L}\right),$$

  where $\mathcal{L}^{-1}$ is the function inverse of the Langevin function $\mathcal{L}(z) \stackrel{\text{def}}{=} \coth z - z^{-1}$.

- Two material parameters:

  – **Rubbery modulus**,

  $$G_0 = N k_B \vartheta,$$

  N — number of chains per unit reference volume.

  – **Network locking stretch**,

  $$\lambda_L = \sqrt{n}$$

  n — number of links (Kuhn-segments) in each polymer chain.

- **Generalized shear modulus**: $G = G_0\left(\dfrac{\lambda_L}{3\bar{\lambda}}\right)\mathcal{L}^{-1}\left(\dfrac{\bar{\lambda}}{\lambda_L}\right)$.

- Since $\mathcal{L}^{-1}(z) \to \infty$ as $z \to 1$, the modulus $G \to \infty$ as $\bar{\lambda} \to \lambda_L$.

  This response is pathological.