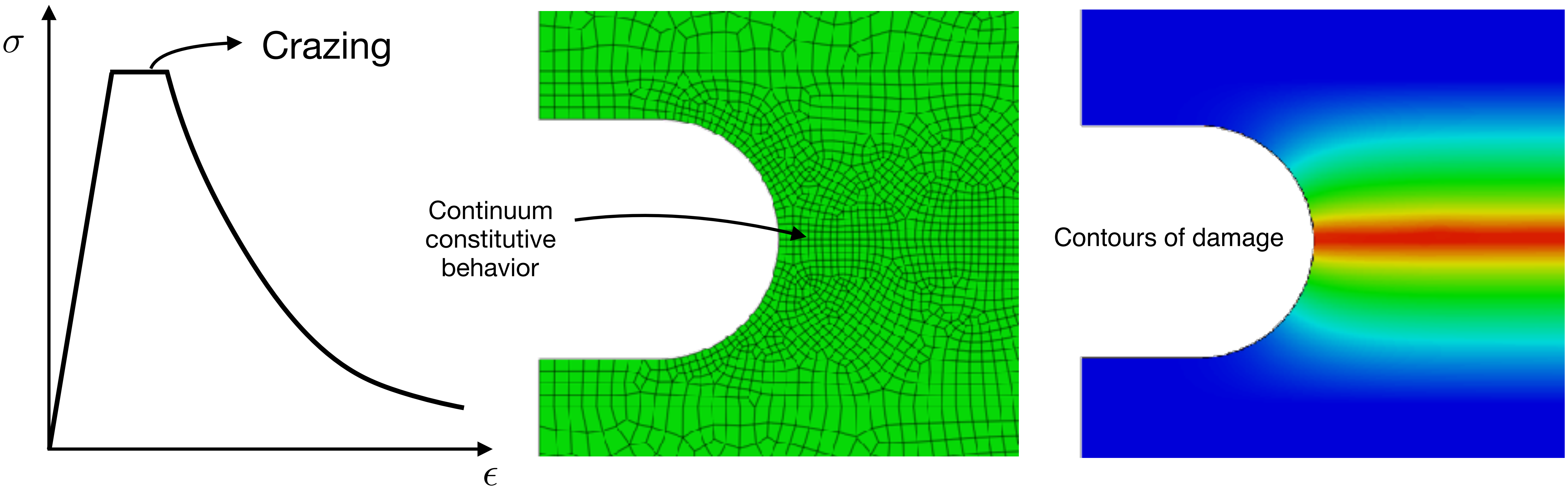
"Crazing" plus gradient-damage theory for quasi-brittle materials



- We introduce some **"craze"-type inelasticity** in the constitutive response of the material, prior to the damage/softening process.

- We use a **gradient-damage theory**.

  - A major reason for using a gradient-damage theory is to **"regularize"** the strain-softening behavior during the damaging process, and to **avoid mesh-dependency related issues during finite element simulations**.

    Cf. the pioneering studies on fracture of quasi-brittle materials (e.g., Bazant,1987; de Borst 1993; Peerlings et al., 1998), and also in the more recent phase-field theories of fracture of brittle materials (de Borst et al., 2016).

Gradient-damage theory

Kinematics

- Motion, deformation gradient, velocity, velocity gradient:

$$\boldsymbol{\chi}(\mathbf{X},t), \qquad \mathbf{F} = \nabla\boldsymbol{\chi}, \qquad \mathbf{v} = \dot{\boldsymbol{\chi}}, \qquad \mathbf{L} = \mathrm{grad}\mathbf{v} = \dot{\mathbf{F}}\mathbf{F}^{-1}.$$

- Multiplicative decomposition of the deformation gradient:

$$\mathbf{F} = \mathbf{F}^e\mathbf{F}^c,$$

  (i) $\mathbf{F}^c(\mathbf{X})$ with $J^c = \det\mathbf{F}^c > 0$ is the **inelastic dilational distortion due to crazing**, and
  (ii) $\mathbf{F}^e(\mathbf{X})$, with $J^e = \det\mathbf{F}^e > 0$ the **elastic distortion**.

- Craze flow:

$$\dot{\mathbf{F}}^c = \mathbf{D}^c\mathbf{F}^c, \qquad \mathbf{D}^c = \dot{\epsilon}^c\mathbf{N}^c, \qquad \text{with} \qquad \dot{\epsilon}^c = |\mathbf{D}^c| \geq 0, \qquad \mathbf{N}^c = \mathbf{m}\otimes\mathbf{m},$$

- Velocity gradient:

$$\mathbf{L} = \mathbf{L}^e + \mathbf{F}^e\mathbf{D}^c\mathbf{F}^{e-1}, \qquad \Longrightarrow \qquad (\nabla\dot{\boldsymbol{\chi}})\mathbf{F}^{-1} = \dot{\mathbf{F}}^e\mathbf{F}^{e-1} + \dot{\epsilon}^c\,\mathbf{F}^e\,\mathbf{N}^c\,\mathbf{F}^{e-1}.$$

- Damage variable,

$$\mathrm{d}(\mathbf{X},t) \in [0,1], \qquad \text{with} \qquad \dot{\mathrm{d}}(\mathbf{X},t) \geq 0.$$

Virtual power formulation of macroscopic and microscopic force balances

Rate-like kinematical descriptors. Internal and External power

$$(\nabla\dot{\boldsymbol{\chi}})\mathbf{F}^{-1} = \dot{\mathbf{F}}^e\mathbf{F}^{e-1} + \dot{\epsilon}^c\,\mathbf{F}^e\,\mathbf{N}^c\,\mathbf{F}^{e-1}, \qquad \text{and} \quad \dot{\mathrm{d}} \geq 0. \qquad (\dagger)$$

- The basic "rate-like" descriptors are the velocity $\dot{\boldsymbol{\chi}}$, the elastic distortion rate $\dot{\mathbf{F}}^e$, the craze strain rate $\dot{\epsilon}^c$, and the damage rate $\dot{\mathrm{d}}$, and these are constrained by (†).
- Internal and external power:

$$\mathcal{W}_{\mathrm{int}}(\mathrm{P}) = \int_{\mathrm{P}} \Big(\mathbf{S}^e : \dot{\mathbf{F}}^e + \pi\dot{\epsilon}^c + \varpi\dot{\mathrm{d}} + \boldsymbol{\xi}\cdot\nabla\dot{\mathrm{d}}\Big)\,dv_{\mathrm{R}},$$

$$\mathcal{W}_{\mathrm{ext}}(\mathrm{P}) = \int_{\partial\mathrm{P}} \mathbf{t}_{\mathrm{R}}(\mathbf{n}_{\mathrm{R}})\cdot\dot{\boldsymbol{\chi}}\,da_{\mathrm{R}} + \int_{\mathrm{P}} \mathbf{b}_{\mathrm{R}}\cdot\dot{\boldsymbol{\chi}}\,dv_{\mathrm{R}} + \int_{\partial\mathrm{P}} \xi(\mathbf{n}_{\mathrm{R}})\,\dot{\mathrm{d}}\,da_{\mathrm{R}}.$$